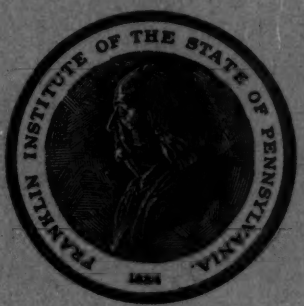
Journal
of
The Franklin Institute

DECEMBER, 1952

1826  1952

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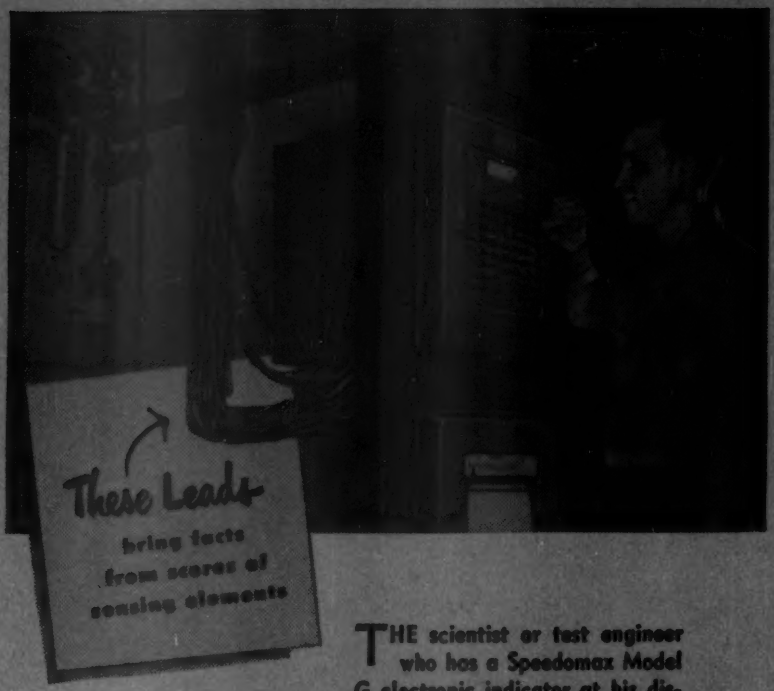

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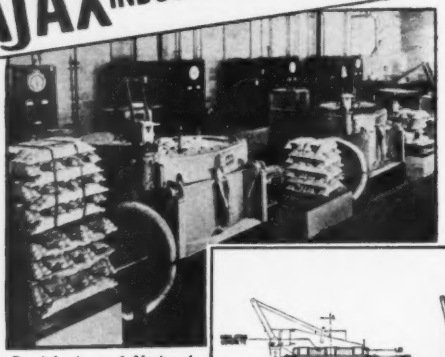
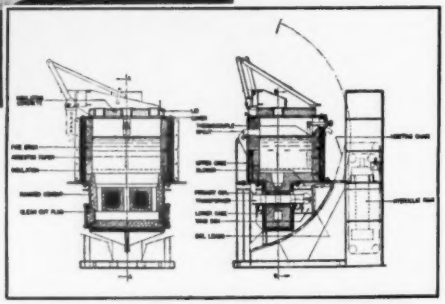
AJAX TAMA-WYATT INDUCTION FURNACES

Cut Foundry Melting Cost

Accurate records at the National Pressure Cooker Co., Eau Claire, Wisc., show that over a two-year period the AJAX Induction Furnace was capable of reducing melting costs by as much as 40%.



Partial view of National Pressure Cooker Co.'s melting room showing AJAX Induction Furnaces.



Cross-section drawings of 166 kW. twin-coil AJAX Induction Furnace with hydraulic tilting device, same as shown in photograph above.

Due to the nature of their use, pressure cooker castings must maintain exceptionally high standards of casting quality. Not only does the induction furnace effect a saving in melting cost but, in addition, working conditions are greatly improved.

Note from the diagram above that the operation of the AJAX Furnace is based on the induction principle, whereby energy is transmitted to the molton charge without actual contact, through the refractory walls. Only the metal is heated, and therefore there are no resistors or other parts having a higher temperature than is absolutely necessary for properly melting the charge. A gentle movement of the bath insures uniform temperature and homogeneous mixing of the ingredients. Linings are made of inert refractories which do not contaminate the melt. Temperature control is entirely automatic.

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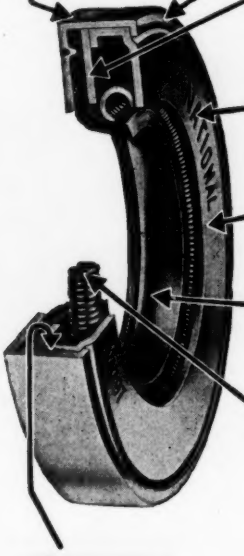

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Performs two functions: Prevents circumferential movement of sealing member in case by clamping same down over indentations protruding into outer case; determines axial position of wiping lip of sealing member at point of contact with shaft.

IDENTIFICATION

Each assembly permanently die-stamped with NATIONAL name and STOCK number for easy and positive identification. No guesswork to determine the correct seal.

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Accurately locates tension-spring and protects sealing member lip; diverts hydraulic impacts; adds further rigidity and strength to outer case while acting as spacer.

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Developed and designed to suit temperature, peripheral and oscillating speeds, pressures, fluids and foreign matter. Accurately formed to proper inside diameter for correct contact with shaft. Formed in steel molds, under hydraulic pressure, to insure consistency and accuracy.

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Performs several functions: Exerts equal tension overall on sealing member wiping lip; takes up wear automatically and holds shape of sealing member constant. Spring tensions vary with shaft diameters. National tensional springs are correctly engineered for each individual shaft size and sealing condition.

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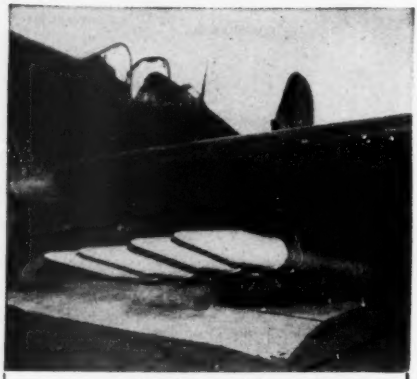

Official Dept. of Defense Photograph

An F4U Corsair with the Navy's new aircraft anti-tank rocket, the "RAM". A Grade I zinc phosphate finish (JAN-C-490) protects the entire external surface of this rocket and provides a durable bond for the specification paint finish.

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The President and the Executive Vice-President are *ex-officio* members of all Committees of the Institute.

New wire spring relay. Older relays had flat metal springs and 70 parts to be handled, compared with 12 in the new model. Relays operate by means of an electromagnet which responds to high-speed pulses.

It splits seconds even faster

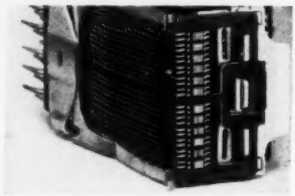
In a split second, relays, which are high-speed switches, set up dial telephone connections. Then they are off to direct the next call. Yet even this speed is too slow for Bell Laboratories scientists in quest of still faster switching.

Scientists and engineers devised a new relay—the wire spring relay—and worked out the production problem with Western Electric, manufacturing unit of the Bell System. This is twice as fast, uses less power and costs less to make and maintain.

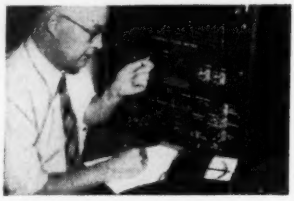
With speedier relays, switching can be done with less equipment . . . and calls go through faster. The wire spring relay is a practical example of how Bell Telephone Laboratories and Western Electric pool their skills to improve telephone service while keeping its cost down.

Bell Telephone Laboratories

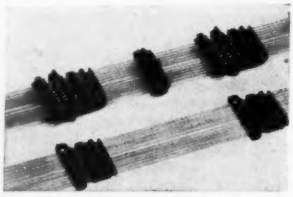
Improving telephone service for America provides careers for creative men in scientific and technical fields.



New relays must be able to operate one billion times—equal to once-a-second for 30 years. Employing a sound recorder as a precision vibrator, Bell scientists learned to evaluate the effect of sideways motion on relay life. Such rubbing motion is limited to one-thousandth of an inch in the new relays.



Dynamic Fluxmeter, developed by Bell Laboratories, indicates flux build-up in intervals of 25 millionths of a second. Precise information like this was essential to higher speed operation.



Relay springs as they come from Western Electric molding machine, before being cut apart for use. Molding technique saves time and money . . . makes possible the maintenance of relays in precision adjustment.

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Vol. 254 **DECEMBER, 1952** **No. 6**

MEDAL DAY PROCEEDINGS

The annual reception, dinner, and presentation of awards, known as "Medal Day," took place at The Franklin Institute on Wednesday, October 15, 1952. The reception began at 6.30 P.M. and dinner was served at 7.30 P.M. Mr. S. Wyman Rolph, President of the Institute, presided at the dinner and presented the subsequent awards.

Reception

The usual reception preceded the dinner and was attended by 360 persons. Immediately following the reception and just before dinner was served in Franklin Hall, Mr. Rolph requested the guests to face the flag above the Franklin statue while Mr. Guy Marriner, director of music at the Institute, played the National Anthem.

Invocation

The Reverend R. Dunham Taylor, Rector of The Memorial Church of The Good Shepherd, delivered the invocation.

Remarks of the President

Mr. Rolph: "My friends, tonight we are gathered to pay such humble tribute as we may, to a group of men who have directed their talents and energies to the investigation and solving of some of our major scientific and industrial problems. The results of their work have contributed greatly to our knowledge and to the greater health, safety and comfort of mankind.

"Those results certainly are in keen observance of the thoughts and philosophy of Benjamin Franklin. Dr. Franklin himself devoted a lifetime to a never tiring quest for new and improved ways of doing things and to a betterment of the standards of his fellows. In all the Medal Days of the past and all that are to come, I feel sure Benjamin Franklin is conscious of our striving and approves it. By this very

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the Journal.)

assembly of men to whom we would do honor, we are honored. As we are honored, so it is appropriate that we should honor the memory of the man who stands as our example. Therefore, I propose a toast, ladies and gentlemen, to the living memory of Benjamin Franklin."

* * *

MR. ROLPH (*continued*): "Honored guests, fellow members of The Franklin Institute and friends:

"On behalf of The Franklin Institute, it is with real pleasure that I, as President, welcome you to this ceremony—the awarding of the medals for 1952 by the Institute. I would like to pause and extend our appreciation to the Hostess Committee of The Franklin Institute for their very thoughtful and able assistance this evening and especially with the floral decorations."

STATED MONTHLY MEETING

MR. ROLPH: "This date, October 15, 1952, marks the first stated monthly meeting of The Franklin Institute since summer recess. The minutes of the last stated meeting have been published in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE. If there are no corrections or additions, I declare them approved as printed.

"It is our custom to elect recipients of the Franklin Medal to Honorary Membership in The Franklin Institute. Dr. Wolfgang Pauli has been so elected. His Certificate of Membership will be presented, *in absentia*, this evening with the Franklin Medal.

"We are proud to have with us tonight many former recipients of the Medals of The Franklin Institute. Mr. Howard Stoertz, Chairman of our Committee on Science and the Arts, will read their names and the award each has received. As he does, will each Medalist rise and remain standing until the full roll is called. I ask your indulgence to withhold your applause until all are standing."

FORMER MEDALIST ATTENDING MEDAL DAY—1952

Year	Medal	Medalist	City
1902	Certificate of Merit	Mr. Myron F. Hill	Westport, Conn.
1907	Longstreth	Dr. Herbert E. Ives	Montclair, N. J.
1915	Longstreth	Dr. Herbert E. Ives	Montclair, N. J.
1919	Longstreth	Dr. Herbert E. Ives	Montclair, N. J.
1911	Longstreth	Dr. Joseph S. Hepburn	Philadelphia, Pa.
1921	Certificate of Merit	Dr. Joseph S. Hepburn	Philadelphia, Pa.
1915	Longstreth	Brig. Gen'l Charles D. Young	Whitford, Pa.
1948	Henderson	Brig. Gen'l Charles D. Young	Whitford, Pa.
1917	Longstreth	Mr. Alan Standish Dana	Ansonia, Conn.
1921	Longstreth	Dr. L. H. Adams	Washington, D. C.
1921	Longstreth	Mr. William B. Eddison	Ardsley-on-Hudson, N. Y.
1922	Longstreth	Mr. Joseph F. Keller	New York, N. Y.
1923	Longstreth	Mr. Harry S. Parks	Pottstown, Pa.
1924	Longstreth	Mr. Thomas C. McBride	Philadelphia, Pa.

1927	Longstreth	Mr. James F. Smathers	Poughkeepsie, N. Y.
1928	Potts	Dr. Eugene C. Sullivan	Corning, N. Y.
1932	Wetherill	Dr. Frank Wenner	Washington, D. C.
1937	Henderson	Dr. Rupen Eksergian	Media, Pa.
1945	Levy	Dr. Rupen Eksergian	Media, Pa.
1937	Longstreth	Mr. Herbert L. Whittemore	Washington, D. C.
1938	Longstreth	Mr. Norman F. S. Russell	Burlington, N. J.
1939	Potts	Dr. Newcomb K. Chaney	Moylan, Pa.
1939	Levy	Prof. Kalman J. DeJuhasz	State College, Pa.
1941	Longstreth	Dr. Benjamin Wilson	Oreland, Pa.
1942	Certificate of Merit	Dr. John J. Grebe	Midland, Mich.
1942	Certificate of Merit	Mr. Walter Larkin	Norristown, Pa.
1945	Certificate of Merit	Mr. Walter J. Coppock	Moylan, Pa.
1946	Levy	Dr. George Southworth	Chatham, N. J.
1947	Ballantine	Dr. George Southworth	Chatham, N. J.
1947	Longstreth	Mr. Samuel Berman	New York, N. Y.
1947	Brown	Mr. Karl P. Billner	Philadelphia, Pa.
1948	Longstreth	Mr. Nicholas F. Arone	Upper Darby, Pa.
1949	Wetherill	Mr. Thomas L. Fawick	Cleveland, Ohio
1950	Clamer	Dr. Charles S. Barrett	Chicago, Ill.
1950	Franklin	Dr. Eugene P. Wigner	Princeton, N. J.
1951	Wetherill	Mr. Reid Berry Gray	Baltimore, Md.
1951	Newcomen	Mr. Isaac Harter	Beaver Falls, Pa.
1951	Certificate of Merit	Mr. John F. McKee	Ardmore, Pa.
1951	Levy	Dr. Albert C. Walker	Short Hills, N. J.

REPRESENTATIVES OF COMPANY MEDALISTS ATTENDING MEDAL DAY—1952

Year	Medal	Company Medalist and City	Represented by
1896	Longstreth	The Pantasote Company New York, N. Y.	Mr. H. W. Wyman
1909	Cresson	Underwood Corporation New York, N. Y.	Mr. D. S. Sammis
1910	Cresson	Automatic Electric Company Chicago, Ill.	Colonel P. G. Black
1916	Cresson	Am. Telephone & Telegraph Co. New York, N. Y.	Mr. P. C. Mabon
1922	Certificate of Merit	Walter Kidde & Company, Inc. Belleville, N. J.	Mr. A. M. Doxsey
1922	Longstreth	Remington Rand, Inc. New York N. Y.	Mr. P. E. Almquist
1927	Wetherill	North East Appliances, Inc. Rochester, N. Y.	Mr. E. A. Halbleib
1932	Wetherill	Monroe Calculating Machine Co. Orange, N. J.	Mr. G. C. Chase
1933	Wetherill	Koppers Company, Inc. Pittsburgh, Pa.	Mr. G. A. Webb
1934	Cresson	Union Switch & Signal Co. Swissvale, Pa.	Mr. N. H. Nicholson
1941	Cresson	United States Navy Washington, D. C.	Rear Admiral J. H. Brown, Jr.
1942	Certificate of Merit	The Goodyear Tire & Rubber Co. Akron, Ohio	Mr. L. E. Spencer
1944	Certificate of Merit	Western Union Telegraph Company New York, N. Y.	Mr. N. P. Corwith

MR. ROLPH (*continued*): "At our stated meetings, it is our custom to call for the Secretary's report. I now call on the Executive Vice-President and Secretary of the Institute, Dr. Henry Butler Allen. Dr. Allen."

REPORT OF THE SECRETARY

"The Institute in 1952"

DR. ALLEN: "Mr. President, Distinguished Guests, Ladies and Gentlemen: It is my pleasant duty on Medal Day to review for our friends the progress at the Institute during the past twelve months. In gathering together some of the highlights I have liked to use as a yardstick the ways in which the Institute has been useful. This is the measure used by our Patron Saint in regulating his entire life. He believed that the best way in which he could thank the Creator for His many blessings was in doing good to his fellow man. And this principle can be applied equally well to institutions as well as to individuals. Also, in telling my little story this evening, I shall try to follow the sage counsel of Franklin when he had Poor Richard say 'Great talkers, little Doers.'

"First of all, following alphabetical sequence, how has our Bartol Research Foundation been useful? The Bartol Laboratory has continued its service in adding to the fund of knowledge about the atom; and also that source of energy coming in from outer space, the cosmic ray. Another machine that produces neutrons—unbelievably minute particles which travel at equally incredible speed, and thus can smash an atom—has been built. These neutron bullets, shot out by the machine, are sprayed against substances, breaking up the atoms of which they are made up. These atom fragments can then be studied. Energy coming from outer space (cosmic rays), a long-time study at Bartol, is being further explored at high altitudes in India.

"The Biochemical Research Laboratory is in somewhat the same situation as the Bartol. It is uncovering useful basic information toward ultimate use—in their case, control or cure of cancer. Cancer results from abnormal growth of the cells out of which body tissue is made. Useful new knowledge has been found about some of the causes of this growth.

"Our Library has now grown to contain 140,000 volumes on science and technology. As our good Librarian says, 'It is a treasure house for researchers.' I can add that you can even find out how to make your clock go, or how to make something with chlorophyll!

"And our Lectures have shown that even with millions of TV and radio sets in homes, people still come to lectures when they can get useful knowledge. Our attendance has grown from an average of 237 to 288.

"We killed two birds with one stone with our 127-year-old JOURNAL OF THE FRANKLIN INSTITUTE. We not only observed the 200th anniversay of Franklin's invention of that most useful device—the lightning rod—but in a way that will be useful to others. We devoted our May issue to four outstanding papers covering the history, the economic significance, and research methods connected with man's efforts to protect himself from lightning. And I am happy to say that later this evening, we will hear from one of the authors.

"One of the useful activities of The Franklin Institute—carried on to a limited extent since our founding in 1824—but which has had the most rapid growth in the past decade—is our Laboratory for Research and Development, dedicated to industrial research. In the past year we have added 40 to our technical staff. We now have 235. Our space is bursting at the seams. Despite our desire to avoid mushroom growth, we have had to expand our space by renting 13,000 square feet across the street. But the useful service that we can render in hydraulics, friction, and physics of metals, forced us to take this step.

"Interestingly enough, one of our research projects in Solid State Physics—that is, knowledge about solid substances so that greater use can be made of them—is in the field for which a medal is being given this evening—a device to replace the vacuum tube, known to all of us who have a radio or TV set. And in the realm of friction, which wastes so much power, we have been given a machine with which we can do research for industry on some of the largest machine bearings (like those on locomotives). In connection with lubrication, as well as friction, we are pleased to have a two-year project for the Association of American Railroads, who are still interested in curing 'hot boxes.' For national defense, the most dramatic news is the '280 mm. atomic cannon,' the design of which has taken us seven years. I witnessed its first public unveiling just a few hours ago at the Aberdeen Proving Ground. It is an achievement for which The Franklin Institute can be justly proud. Our research on asphalt is continuing, supported by the oil producers and a large variety of users.

"The measure of the usefulness of our Museum is the increase in the number of visitors. The attendance in the past year has been the largest in its history except for the opening year when it was a widely-publicized, refreshing novelty. In 1951 more than 250,000 persons visited our Museum and Planetarium. Several of our industrial exhibit supporters have agreed to enlarge or improve their exhibits, notably on petroleum and steel making. We are particularly pleased by our closer relationship with the schools. The schools recognize the inspirational force of our exhibits and our special activities for young people. And a forward step has been undertaken in working out a new exhibit technique. Our Museum Committee has undertaken this experimental study. Our annual Science Fair was again a success. Our

Winter Workshop was again so popular that a series of Scientific Adventures was held during the summer months, and was well attended by young people.

"Our experience with Membership indicates that more and more people are perhaps getting something useful out of belonging to the Institute. At this time last year our enrollment was a bit over 6000. Today it stands at 6440.

"The usefulness of membership to the younger person recently moved one of our long-time members, a manufacturer, to underwrite memberships for 100 boys and girls of high-school graduating age for a five-year term. As the sponsor put it, he—upon going to work out of high school—got a great deal of help through joining The Franklin Institute. He wants to see if that is still so, after 55 years, with young people today.

"A forward step has recently been taken by us—in fact today is its birthday—in establishing at the Institute an Endowment Department in charge of a staff which will more concretely show people how they can give or will their money for a guaranteed useful purpose; and in turn make it possible for the Institute to not only keep out of debt, but to be of more service. For again as Poor Richard said ' 'Tis hard for an empty bag to stand upright.' Even a partly filled bag sags to some extent.

"I can only add that we look forward to a visit from our new, as well as old medalists, and from all our members and friends here as often as you can come to see us. We want to show you what we are continuing to do in our efforts to be useful to all, and get your suggestions. We think you will be interested, too."

MR. ROLPH: "Thank you, Dr. Allen. Ladies and Gentlemen; reports such as you have heard, of the affairs of The Franklin Institute, have been made for the past 128 years.

"The past 128 years have seen many changes in this world and certainly in this nation. There have been wars and there has been peace. We have gone forward and there are times when we seem to have slipped back. Despite these times when we seem to have slipped, there is a key, an identification tag, a word which seems best to fit what has happened. That word is 'Progress.'

"We have moved forward in many fields: socially, politically, in religion, in educational methods and in our economic concepts. I enjoyed the article written by Mr. Joe Livingstone in last night's *Bulletin* telling of the great progress made by agriculture. Industrially, we have leaped forward, thanks to such men as we would honor here tonight. In 128 years we have come from candles to kerosene to fluorescent lighting; from signal fires and wooden semaphores to television; from hand tools to tremendous assembly lines. Yes, we have progressed from a 5 mile per hour locomotive to a 600 mile per hour air transport.

"That is progress, material progress. But we have also progressed in our social concepts. We have put slavery behind us and live in equality of birth and opportunity. We have laid aside the fire and brimstone God of fear and have learned to know more of a God of love.

"There are signs of our being on the threshold of that era of which Benjamin Franklin spoke when he said: 'O that moral science were in a fair way of improvement, that men would cease to be wolves to one another, and that human beings would at length learn what they now improperly call humanity.'

"Let us do honor to men of science and industry here tonight, our Medalists. Before greeting each individual, let me pause for a moment to give due recognition to the men of the Science and Arts Committee of The Franklin Institute, men who devote the entire year to searching and sifting, measuring and balancing, who, among the many, have contributed to our society that which should be recognized. The care and perseverance exercised by the men of this Committee, like their predecessors for the past 100 years, have made the awards to be made here tonight internationally known and regarded with honor in all scientific fields.

"Each Medalist is sponsored by the Chairman of the Sub-Committee of the Science and Arts Committee which recommended him for the award. This Sponsor will read the citation accompanying the Medal.

"Now, we shall call on those Sponsors to present their candidates. Will the Sponsors please take their places?"

PRESENTATION OF THE CERTIFICATE OF MERIT

(Founded in 1882)

A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

(To Arthur M. Stoner, Vice-President, The Jacobs Manufacturing Company, West Hartford, Connecticut.)

MR. ROLPH: "First, the Chair recognizes Mr. F. G. Tatnall."

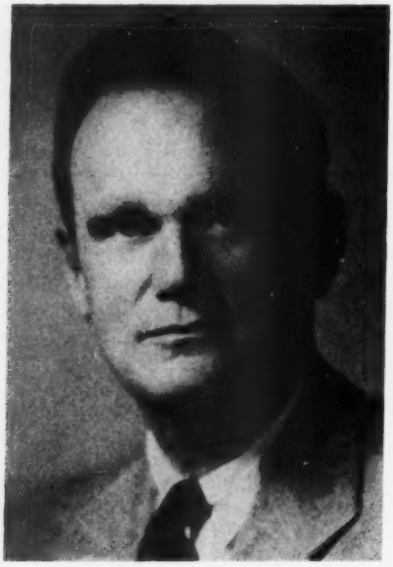
MR. TATNALL: "Mr. President, I present Arthur M. Stoner for an award.

"A collet chuck is intended to hold work that is to be rotated and machined in a lathe. Torque and bending service loads are severe.

"This would seem to be no place for synthetic rubber. Yet, in this spindle nose collet chuck, synthetic rubber is placed in laminations between hardened steel plates which act as a large multiplicity of gripping jaws. The mechanical combination of rubber and steel, steel taking the direct loads, rubber separating the steel plates and providing flexibility and resiliency, provides a very versatile gripping device

which can outperform the conventional and more rigid traditional all metal chuck.

"For instance, the resilient chuck grips of composite materials provide greater accuracy, larger gripping area and higher gripping torque, and accommodate a larger size range of the work which the chuck can grip while maintaining almost perfect parallelism on its long bearing faces. At the same time, it provides an oil and chip seal construction, a minimum of overhang and greater safety in use because when it is closed it is locked. By such a daring departure from traditional practice in machine tool chucking design, the versatility of the chuck is vastly increased. The chuck will hold not only the bright



Arthur M. Stoner

finished metal bars, but, also, resilient and compressible materials such as rubber, plastic and wood, delicate collapsible thin wall tubing and brittle materials such as ceramics and glass. This is where the large number of extra long collet jaw segments, held in perfect parallelism by the flexible rubber segments, evenly distributes the gripping area around the part being held rather than concentrating it in three or four positions of high stress.

"This is a badly needed advancement in the machine tool art, which comes at a time when it can be of greatest service.

"I present Arthur M. Stoner, of Hartford, Connecticut, as a candidate for a Certificate of Merit, 'In consideration of his achievement in producing outstanding advantages in the development of machine tool

chucks resulting in an improved device known as the Spindle Nose Lathe Collet Chuck.' "

MR. ROLPH: "Mr. Stoner, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to you this Certificate of Merit and the Report which accompanies it."

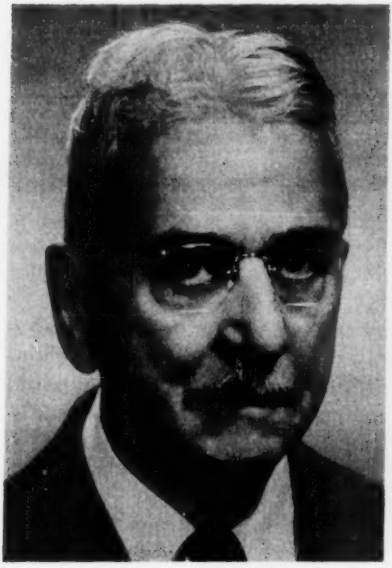
MR. STONER: "Thank you."

MR. ROLPH: "May I call your attention, ladies and gentlemen, to an exhibit of the Collet Chuck referred to in the area just outside this Hall."

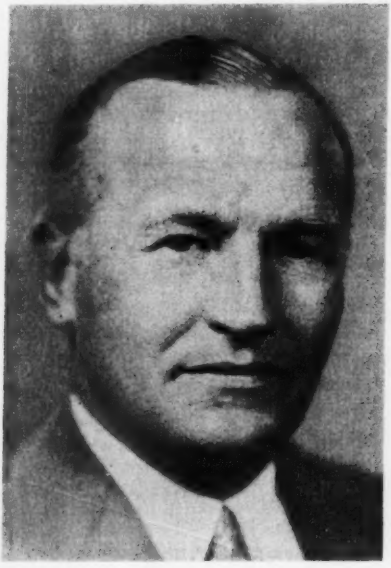
PRESENTATION OF THE ELLIOTT CRESSON MEDALS

(Founded in 1848)

This Medal is awarded for the discovery or original research which adds to the sum of human knowledge, not taking into account commerical values.



Edward C. Molina



H. Birchard Taylor

(To Edward C. Molina, Telephone Engineer, East Orange, New Jersey; and to H. Birchard Taylor, Engineer, Industrialist and Author, Haverford, Pennsylvania.)

MR. ROLPH: "The Chair recognizes Mr. Alan C. Byers."

MR. BYERS: "Mr. President, I present Edward C. Molina for an award.

"Our modern civilization, characterized by many specialized technologies, is knit together by an efficient web of communication networks, without which our present state could not have been achieved. The development of these networks to their present form, and their continued efficient expansion are greatly dependent upon the scientific work of Mr. Molina. His inventions of switching apparatus have given rise to new concepts of telephone switching equipment, and his mathematical developments have given us new techniques of forecasting telephone traffic demands. He has found effective ways of using mathematical probability theory in this work, and has created new tools of analysis, which other workers in other fields have found valuable.

"I present Edward C. Molina, of East Orange, New Jersey, as a candidate for an Elliott Cresson Medal, 'For his important contributions to the mathematical theory of probability and for his significant contributions to the improvement of telephonic communications both by the application of mathematical probability to the study of telephone traffic and by the invention of switching equipment.' "

MR. ROLPH: "By the same authority, Mr. Molina, I present to you this Elliott Cresson Medal and the Certificate and Report which accompany it."

MR. MOLINA: "Thank you."

MR. ROLPH: "I think little need be said at this time of the utter and absolute dependence of this nation upon its telephonic communication systems. That Mr. Molina has contributed so much to that system should and has been duly recognized.

"The Chair now recognizes Professor W. S. Pardoe."

PROFESSOR PARDOE: "Mr. President, I present H. Birchard Taylor for an award.

"Mr. Taylor was born in Philadelphia, educated in our high schools and the University of Pennsylvania. He was employed throughout his active career by the I. P. Morris Division of the William Cramp and Sons Engine & Shipbuilding Company (afterward the Baldwin-Southwark Company) as draftsman, engineer, chief engineer and engineer in charge of sales and development of some of the largest single runner, vertical hydraulic turbines built in these United States and Canada.

"He is an inventor with a large number of patents, a public servant, a Trustee of his Alma Mater, and a prolific producer of profound papers on technical and philosophical subjects read before engineering and other societies both here and abroad.

"I present H. Birchard Taylor, of Philadelphia, Pennsylvania, as a candidate for an Elliott Cresson Medal, 'In consideration of his inspiring leadership in the development of the single runner vertical reaction turbine, his many patents and his contributions to technical literature.' "

MR. ROLPH: "Mr. Taylor, by the same authority, I present to you

this Elliott Cresson Medal and the Certificate and Report which accompany it."

MR. TAYLOR: "Thank you."

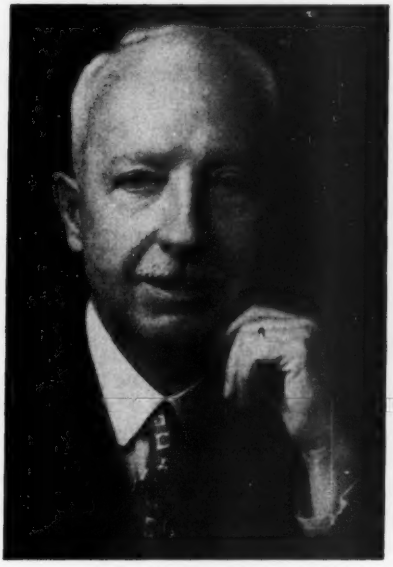
MR. ROLPH: "Birchard Taylor's contribution to the greatly expanded electric power facilities of this country and Canada by his development of the single runner vertical reaction turbine is in itself noteworthy. The value of Mr. Taylor as a consultant and colleague in the Navy Department has long been a matter of record—for without his counsel and assistance, that part of the war effort which concerned itself with the rapid production of ships and their necessary components would have been needlessly held up."

PRESENTATION OF THE GEORGE R. HENDERSON MEDAL

(Founded in 1924)

This Medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

(To Alfred J. Büchi, Engineer, Winterthur, Switzerland.)



Alfred J. Büchi

MR. ROLPH: "The Chair recognizes Dr. Rupen Eksergian."

DR. EKSERGIAN: "Mr. President, I present Alfred J. Büchi for an award.

"A principal contribution to modern Diesel Engine Design, par-

ticularly in the locomotive field, is the incorporation of the turbine supercharger, which utilizes the exhaust gases of the engine to drive a compressor for precompressing the intake air. In the 'Büchi system,' the exhaust pipe between engine and turbine is specifically designed to induce pressure fluctuations, resulting in improved scavenging and augmented compression. The exhaust pipe can be regarded as an acoustic bottle where the gases in the cylinder represent the elastic medium and the exhaust pipe represents the inertia element of the flow. By timing this system, consistent with the period of the engine rotation, a suction effect is obtained in the cylinder after the gas is accelerated along the pipe during the exhaust period. This results in a lowered pressure at the beginning of scavenging, in increasing the volumetric efficiency for the intake air, and improved cooling of the cylinder. The inertia of the exhaust gases, in turn, maintains a more uniform torque at the turbine, thereby augmenting the compression during the initial air intake to the cylinder. The 'Büchi system' requires special combination of piping to the cylinders in order to carry out this operation without interference of the exhaust gases. On the basis of this system, consisting of special combinations of piping to multi-cylinder engines, along with a vastly improved design of compressor and turbine, resulting in high efficiencies, the 'Büchi system' for supercharging four-cycle Diesel Engines has augmented the power from 30 to 50 per cent, and, in special designs, over 100 per cent. The 'Büchi system' for supercharging is universally recognized as a major component to the modern Diesel Engine where, in the locomotive field, we find its application extending to over 7,000,000 h.p., and, for other classes, to over 15,000,000 h.p.

"I present Alfred J. Büchi, of Winterthur, Switzerland, as a candidate for the George R. Henderson Medal, 'In view of his contributions in augmenting the capacity and performance of the four-cycle Diesel Engine through his supercharging system in all classes of service, but particularly in the railway field.' "

MR. ROLPH: "Dr. Büchi, by the same authority, I present to you this George R. Henderson Medal and the Certificate and Report which accompany it."

DR. BÜCHI: "Thank you."

PRESENTATION OF THE JOHN PRICE WETHERILL MEDALS

(Founded in 1925)

This Medal is awarded for discovery or inventions in the physical sciences or for new and important combinations of principles or methods already known.

(To Harrison P. Hood and Martin E. Nordberg, Research Chemists, Corning Glass Works, Corning, New York; and Albert J. Williams, Jr.,

Associate Director of Research, Leeds & Northrup Company, Philadelphia, Pennsylvania.)

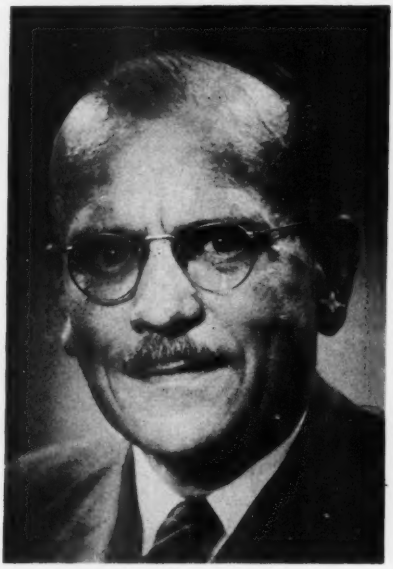
MR. ROLPH: "The Chair recognizes Dr. William G. Schmidt."

DR. SCHMIDT: "Mr. President, I present Harrison P. Hood and Martin E. Nordberg for awards.

"In their painstaking, detailed, and well-organized researches, they have developed a series of high silica glasses which are economically and specifically adapted to limited, though important, areas of scientific endeavor.

"A highly siliceous glass responds to a vital scientific need by virtue of its properties of exceptional stability, high softening point and an

Harrison P. Hood



Martin E. Nordberg

extremely low coefficient of thermal expansion. In sintering and calcining operations, in interactions involving acids, mild alkalies, water and steam at high temperatures, there is no replacement for these silica glasses except vessels or ducts of pure quartz which is frequently prohibitive because of its relatively excessive cost. Like quartz, these high silica glasses are transparent to ultra-violet light of 250 millimicrons or less and are receiving industrial application accordingly. A few of the specially designed glasses are freely permeable to the near infra-red wave lengths as well.

"The methods employed by the inventors represent the penetrating and comprehensive insight into the intricacies of a scientific problem,

the embracing sweep of its many variables and intangibles, and the consummate control that distinguish every prudently planned and ably executed piece of research.

"Without detailing the technical details, their researches in glass making involve the successive steps of producing a homogeneous melt, of a controlled heat treatment yielding two phases—one of silica and the other of boric oxide and alkali—of an acid immersion leaving the silica phase intact, of leaching away the soluble phase, and of finally vitrifying and dehydrating the cellular structure into a desirable vessel of high silica content.

"I present Harrison P. Hood and Martin E. Nordberg, both of Corning, New York, as candidates for John Price Wetherill Medals, 'In consideration of the discovery of a truly novel process for the manufacture of unusual, high silica glasses comprising the vitrification of an insoluble phase of a heat-treated alkali-borosilicate glass after its soluble phase has been leached with inorganic acid.' "

MR. ROLPH: "By the same authority, I present to each of you these John Price Wetherill Medals and the Certificates and Reports which accompany them."

MR. HOOD: "Thank you."

DR. NORDBERG: "Thank you."

MR. ROLPH: "The Chair recognizes Mr. J. Philip Evans."

MR. EVANS: "Mr. President, I present Albert J. Williams, Jr., for an award.

Albert J. Williams, Jr.

"Mr. Williams is Associate Director of Research, Leeds & Northrup Company, Philadelphia, Pennsylvania. During the past twenty-five years, he has been actively engaged in the field of instrumentation, and during that time he has directed numerous research projects.

"His broad experience and abilities in the field of electronic research have resulted in numerous contributions in this complex field of science. He is the co-author of twenty-one patents, six of which are on recording instruments and two directly related to recorders.

"His invention of the Speedomax was the result of painstaking efforts to attain speeds of action in recorders, as required by many problems in numerous types of industry.

"This invention has made possible the control and recording of process variables which heretofore have been unattainable. It has opened new fields of research and has made possible new scientific measurements which, in turn, have contributed to other technical investigations.

"I present Albert J. Williams, Jr., of Philadelphia, Pennsylvania, as a candidate for a John Price Wetherill Medal, 'In recognition of his invention of the Speedomax, incorporating for the first time an electronic balance detector.' "

MR. ROLPH: "Mr. Williams, by the same authority, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it."

MR. WILLIAMS: "Thank you."

MR. ROLPH: "During the earlier part of the evening, I am sure you all had an opportunity to see in action the Speedomax electronic balance detector for which Mr. Williams is being recognized. The particular test shown is only one of the many applications of such a device."

PRESENTATION OF THE WALTON CLARK MEDAL

(Founded in 1926)

> This Medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

(To Edwin Loy Hall, Director, Laboratories, American Gas Association, Inc., Cleveland, Ohio.)

MR. ROLPH: "The Chair recognizes Mr. W. H. Fulweiler."

MR. FULWEILER: "Mr. President, I present Edwin Loy Hall for an award.

"The extension of natural gas lines to the densely populated cities along the East Coast, where an interruption of the supply would be a

catastrophe, has made the problem of insuring the continuity of supply of vital importance since the wells may be thousands of miles away and the pipe line subject to potential sources of interruption.

"Geological conditions, in many situations, limit the development of adequate underground storage reservoirs. Consequently, the production of an auxiliary supply of suitable quality as a substitute for, or in order to supplement the existing supply, has become of great importance.

"Present processes involve the utilization of petroleum products that can be stored in large quantities.

Edwin Loy Hall

"Mr. Hall has devoted much of his business life to the study of the transformation of hydrocarbons. His earlier work involved the improvement of existing processes and the transformation of hydrocarbons having a low form value, to more valuable types. His most important work, in view of the present situation, has been the development of a process for production of a high heating value gas suitable for use as a substitute for natural gas.

"He is presently in charge of the Testing Laboratories of the American Gas Association, engaged in important fundamental research on the utilization of gas for industrial and domestic purposes.

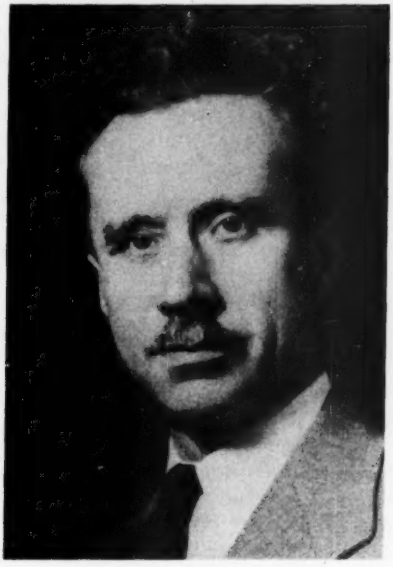
"I present Edwin Loy Hall, of Cleveland, Ohio, as a candidate for the Walton Clark Medal, 'In consideration of his many contributions to

the gas industry, in the improvement of old processes for gas manufacture, for the development of new processes for the production of valuable by-products by the pyrolysis of petroleum oils, and for the invention of a new and more efficient process for the production of high heating value gas as a substitute for natural gas.' "

MR. ROLPH: "Mr. Hall, by the same authority, I present to you this Walton Clark Medal and the Certificate and Report which accompany it."

MR. HALL: "Thank you."

MR. ROLPH: "In the outer lobby, there is an excellent graphic illustration of those improvements in gas manufacturing processes for which we are saluting Mr. Hall this evening. If you will note the number of patents to the credit of Mr. Hall, you will perhaps gain some appreciation of the man we are honoring."



Fred N. Severud

PRESENTATION OF THE FRANK P. BROWN MEDAL

(Founded 1938)

This Medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

(To Fred N. Severud, Consulting Engineer, New York, New York.)

MR. ROLPH: "The Chair now recognizes Mr. Coleman Sellers, III."

MR. SELLERS: "Mr. President, I present Fred N. Severud for an award.

"Our candidate has had a remarkable engineering career in the field of building construction. He has shown that he has vision and resourcefulness in his designs which are always sound and carefully calculated. Rarely does one find any one man with such a diversified record of brilliant accomplishments.

"In a large housing development he designed in the 30's, he introduced advanced methods of wall and floor construction which have since won him wide acclaim. In recent years, he has been engaged in consulting work on hospitals, research laboratories, airports, theaters, housing projects, television development and hangars as well as all types of industrial buildings and marine construction.

"Besides his engineering practice, he has served as technical advisor to the Norwegian Government and consultant to other foreign countries.

"For the United States Government, he has engineered many projects for the Atomic Energy Commission, the Army, Navy, Coast Guard, Maritime Commission, Department of State, and Veterans' Administration.

"Mr. Severud is the author of many technical articles and is the co-author of two books, one on Apartment Houses and the other on the effect of atomic blast on structures. The latter is the result of exhaustive studies which he made.

"I present Fred N. Severud, of New York, New York, as a candidate for the Frank P. Brown Medal, 'In consideration of his outstanding engineering accomplishments in the field of building construction, characterized by intelligent thinking, whereby he has combined sound engineering principles with ingenious and novel designs to produce improvements giving simplicity, economy, usefulness and pleasing effects.' "

MR. ROLPH: "Mr. Severud, by the same authority, I present to you this Frank P. Brown Medal and the Certificate and Report which accompany it."

MR. SEVERUD: "Thank you."

MR. ROLPH: "To list all of Mr. Severud's achievements in the field of design and building construction would take the better part of this evening. May I invite your attention, if you have not already seen them, to a partial listing of those achievements beneath Mr. Severud's picture in the outer lobby."

PRESENTATION OF THE FRANCIS J. CLAMER MEDAL

(Founded in 1943)

This Medal is awarded at least once in five years for meritorious achievement in the Field of Metallurgy.

(To Cyril Stanley Smith, Professor of Metallurgy, and Director of the Institute for the Study of Metals, The University of Chicago, Chicago, Ill.)

Cyril Stanley Smith

MR. ROLPH: "The Chair recognizes Mr. Francis B. Foley."

MR. FOLEY: "Mr. President, I present Cyril Stanley Smith for an award.

"Details of the work of a number of our scientists, bearing on the development of energy by nuclear fission during the past decade, still remain secret for well understood reasons of national security. This is true of noteworthy metallurgical contributions of Dr. Smith.

"There has been found in nature only one element, uranium 235, suitable for the release of energy by nuclear fission. Because of its scarcity, scientists were led to develop methods for the production, in a man-made environment, of a new element, plutonium. The metallurgical characteristics of plutonium, an element previously unknown, had to be determined if progress towards its utilization as a source of nuclear energy was to be made. The metallurgical knowledge and experience and the technical direction of Dr. Smith were called upon and, despite the necessity of having to work with minute quantities of material and under the handicap of certain health hazards, the necessary information was made available.

"I present Cyril Stanley Smith, of Chicago, Illinois, as a candidate for the Francis J. Clamer Medal, 'In recognition of his work leading to

knowledge of the basic factors of the metallurgical behavior of elemental plutonium essential to the development of nuclear energy.' "

MR. ROLPH: "Dr. Smith, by the same authority, I present to you this Francis J. Clamer Medal and the Certificate and Report which accompany it."

DR. SMITH: "Thank you."

MR. ROLPH: "There is much that could be said of Dr. Smith's work with the Atomic Energy Commission, but much of it is still classified. I believe it is only necessary to note that our nation would perhaps not hold the position of dominance in the world which it does and that the advances in the use of atomic energy in medical research would not have been realized without such contributions."

PRESENTATION OF THE STUART BALLANTINE MEDALS

(Founded in 1946)

This Medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

(To John Bardeen, Professor of Physics and Electrical Engineering, University of Illinois, Urbana, Illinois, and Walter H. Brattain, Research Physicist, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey.)

John Bardeen

Walter H. Brattain

MR. ROLPH: "The Chair recognizes Dr. Winthrop Wright."

Dr. WRIGHT: "Mr. President, I present John Bardeen and Walter H. Brattain for awards.

"The introduction by Lee DeForest in 1907 of a third electrode in the Fleming Valve led directly to the development of vacuum electronic tubes which now are primary constituents of almost all circuits used in communications and reconnaissance.

"The crystal rectifier came into wide use during the early years of the last decade. These devices require neither a heated cathode nor an evacuated envelope but their performance closely parallels that of the Fleming Valve. The possibility of adding a third electrode to the crystal diode, thus obtaining the electrical equivalent of DeForest's audion, occurred to many investigators. In 1948 our Medalists, both of whom were then associated with the Bell Telephone Laboratories, announced the invention of the Transistor and demonstrated its use in an amplifying circuit. Their first transistors, like the early audions, were crude and unreliable, but their achievement ranks high in the annals of the electronic art.

"The transistor, whether of the point contact type invented by Bardeen and Brattain or of the junction type invented by others, has passed the laboratory stage. It is finding successful applications in telephone circuits, in hearing aids, in high speed computers and in reconnaissance devices of the armed services.

"I present John Bardeen, of Urbana, Illinois, and Walter H. Brattain of Norristown, New Jersey, to each of whom has been awarded a Stuart Ballantine Medal, 'In consideration of their contributions to the theory of surface states in semi-conductors and of their invention of the Point Contact Transistor, a device foreshadowing a notable advance in the means of electromagnetic communications.' "

MR. ROLPH: "Dr. Bardeen, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you the Stuart Ballantine Medal and the Certificate and Report which go with it."

DR. BARDEEN: "Thank you."

MR. ROLPH: "Dr. Brattain, by that same authority, I present to you the Stuart Ballantine Medal and the Certificate and Report which accompany it."

Dr. Brattain: "Thank you."

MR. ROLPH: "As in the case of Mr. Molina, here are two men who have played such an important part in the tremendous development of electromagnetic communications that it is almost impossible at this time to reckon their contributions to our society. In the Bell Telephone Room of the Museum, the Transistors are on exhibit. For those of you who do not understand its use, I recommend a visit to that room."

PRESENTATION OF THE FRANKLIN MEDAL

(Founded in 1914)

This Medal is awarded annually from the Franklin Medal Fund, founded January 1, 1914, to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

(To Wolfgang Pauli, Physikalisches Institut Polytechnicum, Zurich, Switzerland.)

Wolfgang Pauli

MR. ROLPH: "Tonight, it is a matter of extreme regret that the recipient of the Franklin Medal cannot be with us. In the absence of Dr. Wolfgang Pauli of Zurich, Switzerland, whose duties made it necessary for him to remain in Switzerland at this time, Mr. Maurice J. Rohrbach, Consul for Switzerland in Philadelphia, will stand as proxy.

"The Chair now recognizes Dr. Pauli's Sponsor, Mr. Charles H. Masland, II, who will present his candidate for the Franklin Medal."

MR. MASLAND: "Mr. President, I present, *in absentia*, Wolfgang Pauli for an award.

"Dr. Pauli's exclusion principle is one of the more important pieces of mankind's knowledge of the physical world. It deals with the

states of motion of a number of the small particles of which atoms are made. This principle applies to protons, to neutrons, to electrons, but it does not apply to photons, the particles of light.

"These particles are so small and they can move so rapidly that it has not been possible to set up satisfactory physical models describing their motions. Their motions are best described in terms of mathematical probabilities concerned with their velocities and positions and momenta. Dr. Pauli's principle states that only certain types of mathematical functions can be used for describing the state of motion of these particles. States corresponding to other types of functions are excluded.

"The Pauli principle has been particularly useful in defining the types of motion possible for electrons associated with atomic nuclei.

"For many years, knowledge of chemical valence had been largely on an empirical basis. Pauli's principle has provided a firm theoretical base upon which a much sounder knowledge of valence has been built. His principle has provided the key to understanding the arrangement of the elements in the periodic system. It has acted as a guide in much of the modern work in magnetism, in the theory of solids and in atomic spectroscopy. Pauli's exclusion principle has taken the position of a general law within its field.

"I present, *in absentia*, Wolfgang Pauli, of Zurich, Switzerland, as a candidate for the Franklin Medal, 'In consideration of his work toward the understanding of atomic physics and specifically for his formulation of the exclusion principle.' "

MR. ROLPH: 'By the authority vested in me as President of The Franklin Institute of the State of Pennsylvania, I present, *in absentia*, to Dr. Wolfgang Pauli, through you, Mr. Rohrbach, this Franklin Medal with its accompanying Certificate and Report, and a Certificate of Honorary Membership in The Franklin Institute."

MR. ROHRBACH: "Thank you."

MR. ROLPH: "The sponsors may now resume their places."

* * *

MR. ROLPH (*continued*): "It is now my pleasure to present to you Dr. I. Bernard Cohen, of the Department of the History of Science at Harvard University. Dr. Cohen, in addition to being Managing Editor of *Isis*, a most famous technical publication, is internationally known as the authority on the scientific achievements of Benjamin Franklin. In the JOURNAL OF THE FRANKLIN INSTITUTE for May this year, which was published in commemoration of the 200th anniversay of Franklin's 'Philadelphia Experiment,' Dr. Cohen contributed a most interesting paper entitled 'Prejudice Against the Introduction of Lightning Rods.' It may be that we have much to learn of Franklin and his lightning rods, so it is with real pleasure that I introduce Dr. Cohen."

* * *

Dr. Cohen spoke with accomplished ease and assurance, covering not only the lightning rod, but also Franklin's position as a scientist in the 17th century, both in his own country and abroad. Since this is one of the few subjects relating to Franklin which has not been exhaustively treated in the literature, Dr. Cohen's remarks were doubly appreciated.

After Dr. Cohen had delivered his talk, Mr. Rolph conveyed to him the thanks of The Institute, dismissed the assembly with a cordial word and declared the meeting adjourned.

EDITOR'S NOTE: It is a matter of regret that, since Dr. Cohen spoke extemporaneously, no manuscript of his talk is available for publication.

ANALYSIS OF PARTLY SYMMETRICAL MACHINES BY MEANS OF UNITARY TRANSFORMATION *

BY

D. W. C. SHEN [1] AND H. N. G. BROADBENT [1]

SUMMARY

The instantaneous impedance matrices of machines having electric and magnetic circuits on but one side of the air gap symmetrically arranged are shown to be Hermitian and unitary. Unitary matrices containing variable base vectors are used to transform the operational equations relating voltages and currents in six circuits, *viz.*, three stationary armature coils and three rotating field coils on salient pole. The transformed equation is compact, convenient and amenable to transient solutions. It is correlated to previous results obtained by Park's transformation in a simple manner.

An analysis is then made on asynchronous operation under balanced terminal voltages with the main field short circuited. The results include the well-known Georges phenomenon of induction motor with single phase rotor.

PARTIALLY-SYMMETRICAL MACHINES

Polyphase machines may be classified according to their magnetic as well as their electrical symmetry of construction; the problem may be readily grouped into two classes. In the case of completely symmetrical machines, the polyphase windings on both sides of the air-gap are symmetrically arranged and the magnetic permeance is independent of the relative positions of the stator and the rotor. The so-called symmetrical alternator or motor studied by Rudenberg (1) [2] is simply a wound-rotor induction machine running at synchronous speed and excited by direct current. The partially symmetrical machines, the circuits on but one side of the air-gap being symmetrical, may be subdivided into round-rotor and salient-pole machines if only steady-state balanced operation is to be considered. In transient analysis, however, the most general case would be a salient-pole machine with three separate rotor windings, two in the direction of the polar axis and one in the direction of the inter-polar axis. It may be noted that when unbalanced condition is to be discussed, only the completely symmetrical machine will be free from harmonics.

INSTANTANEOUS IMPEDANCE MATRICES

The instantaneous self-impedance matrix of the stator of a partially symmetrical machine Z_{aa} may be written as two parts: one being the

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

constant part is of the usual form for a completely symmetrical stationary network.

$$\mathbf{Z}_{a0} = (\gamma_a + L_a p)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M_{a0}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

γ_a = resistance of stator per phase to neutral,

$L_a = L_{a0} + M_{a0}$ = constant part of the self-inductance of stator per phase (L_{a0}) plus the constant part of the mutual inductances between two phases (M_{a0}),

$P = \dfrac{d}{dt}$ = the time differential operator.

The variable part being a second harmonic function of time characterizes the saliency feature of the machine and is simply

$$\mathbf{Z}_{a2} = p\begin{bmatrix} L_{a2}\cos 2\theta & M_{a2}\cos 2(\theta+120) & M_{a2}\cos 2(\theta-120) \\ M_{a2}\cos 2(\theta+120) & L_{a2}\cos 2(\theta-120) & M_{a2}\cos 2\theta \\ M_{a2}\cos 2(\theta-120) & M_{a2}\cos 2\theta & L_{a2}\cos 2(\theta+120) \end{bmatrix} \tag{2}$$

where

L_{a2} = maximum value of the second harmonic of the self-inductance of armature per phase,

M_{a2} = maximum value of the second harmonic of the mutual inductance between two phases,

$\theta = \nu t$, ν being the rotational angular velocity between the armature and the field windings.

In terms of the following two matrices,

$$\mathbf{H} = \begin{bmatrix} 1 & a & a^2 \\ 1 & a & a^2 \\ 1 & a & a^2 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 1 & a^2 & a \\ a^2 & a & 1 \\ a & 1 & a^2 \end{bmatrix}$$

$$a = \epsilon^{j\frac{2\pi}{3}} \qquad a^2 = \epsilon^{-j\frac{2\pi}{3}}.$$

$\mathbf{Z}_{a2}$ may be expressed as

$$\mathbf{Z}_{a2} = \frac{p(L_{a2} - M_{a2})}{6}\epsilon^{j2\theta}(\mathbf{H} + \mathbf{H}_t) + \frac{p(L_{a2} - M_{a2})}{6}\epsilon^{-2j\theta}(\mathbf{H}^* + \mathbf{H}_t^*)$$
$$+ p\left(\frac{L_{a2}\epsilon^{j2\theta} + M_{a2}\epsilon^{-j2\theta}}{6}\right)\mathbf{G} + p\left(\frac{L_{a2}\epsilon^{-j2\theta} + M_{a2}\epsilon^{j2\theta}}{6}\right)\mathbf{G}^*. \tag{3}$$

In the above, the asterisk (*) denotes the conjugate and the subscript, t, means the transpose of the matrix.

As for the armature-reaction matrix, it may be noted that $\mathbf{M}_{af}$, the mutual-inductance matrix of the rotor with respect to the stator, is the transpose of $\mathbf{M}_{fa}$, the mutual-inductance matrix of the stator with respect to the rotor. Thus,

$$p\mathbf{M}_{fa} = p\begin{bmatrix} M_{1a}\cos\theta & M_{1a}\cos(\theta-120) & M_{1a}\cos(\theta+120) \\ M_{2a}\cos\theta & M_{2a}\cos(\theta-120) & M_{2a}\cos(\theta+120) \\ -M_{3a}\sin\theta & -M_{3a}\sin(\theta-120) & -M_{3a}\sin(\theta+120) \end{bmatrix}$$

$$= \tfrac{1}{2}p\begin{bmatrix} M_{1a} & a^2M_{1a} & jaM_{1a} \\ M_{2a} & a^2M_{2a} & jaM_{2a} \\ M_{3a} & a^2M_{3a} & jaM_{3a} \end{bmatrix}\epsilon^{j\theta} + \tfrac{1}{2}p\begin{bmatrix} M_{1a} & aM_{1a} & -ja^2M_{1a} \\ M_{2a} & aM_{2a} & -ja^2M_{2a} \\ M_{3a} & aM_{3a} & -ja^2M_{3a} \end{bmatrix}\epsilon^{-j\theta} \quad (4)$$

in which M_{1a}, M_{2a}, M_{3a} are the maximum values of mutual inductances between one armature phase and the following: the main field winding, the second winding on the direct axis, and the winding along the quadrature axis respectively.

Let the resistances and self-inductances of the two direct-axis field windings and the one quadrature-axis winding be r_1, r_2, r_3 and L_1, L_2, L_3, respectively. Also let the mutual inductance between the two coincident-axis windings be M_{12}; then the self-admittance matrix for the rotor is

$$\mathbf{Y}_{ff} = \begin{bmatrix} r_1 + L_1p & M_{12}p & 0 \\ M_{12}p & r_2 + L_2p & 0 \\ 0 & 0 & r_3 + L_3p \end{bmatrix}^{-1}. \quad (5)$$

By performing the operation, $p\mathbf{M}_{af}\cdot\mathbf{Y}_{ff}\cdot p\mathbf{M}_{fa}$, the armature-reaction matrix can be decomposed into the following Hermitian and unitary matrices:

$$A(p)\begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix} + A^*(p)\begin{bmatrix} 1 & a^2 & a \\ a & 1 & a^2 \\ a^2 & a & 1 \end{bmatrix}$$

$$+ B(p)\begin{bmatrix} 1 & a^2 & a \\ a^2 & a & 1 \\ a & 1 & a^2 \end{bmatrix}\epsilon^{j2\theta} + B^*(p)\begin{bmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{bmatrix}\epsilon^{-j2\theta}. \quad (6)$$

$A(p)$ and $B(p)$ together with their conjugates are operational functions (see Appendix).

The instantaneous impedance matrix of the machine is $\mathbf{Z} = \mathbf{Z}_{aa} - p\mathbf{M}_{af}\cdot\mathbf{Y}_{ff}\cdot p\mathbf{M}_{fa}$, and from Eqs. 1, 2, and 3, may be put into the

following form:

$$\mathbf{Z} = \frac{\gamma_a + p(L_{a0} - 2M_{a0})}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} + \frac{\gamma_a + pL_a - 3A(p)}{3}\begin{bmatrix}1 & a & a^2\\ a^2 & 1 & a\\ a & a^2 & 1\end{bmatrix}$$

$$+ \frac{\gamma_a + pL_a - 3A^*(p)}{3}\begin{bmatrix}1 & a^2 & a\\ a & 1 & a^2\\ a^2 & a & 1\end{bmatrix} + \frac{p(L_{a2} - M_{a2})}{6}\epsilon^{2j\theta}(\mathbf{H} + \mathbf{H}_t)$$

$$+ \frac{p(L_{a2} - M_{a2})}{6}\epsilon^{-j2\theta}(\mathbf{H}^* + \mathbf{H}_t^*)$$

$$+ \frac{\frac{1}{2}L_{a2}p + M_{a2}p - 3B(p)}{3}\epsilon^{j2\theta}\mathbf{G}$$

$$+ \frac{\frac{1}{2}L_{a2}p + M_{a2}p - 3B^*(p)}{3}\epsilon^{-j2\theta}\mathbf{G}^*. \qquad (7)$$

Denoting the emf. due to d-c. excitation by $\mathbf{E}_a$, which is equal to $-p\mathbf{M}_{af}\cdot I_{dc}$, the voltage equation of a partially-symmetrical machine is

$$\mathbf{V}_a = \mathbf{E}_a + \mathbf{Z}\cdot\mathbf{I}_a \qquad (8)$$

where

$$\mathbf{V}_a = \begin{bmatrix}V_a\\ V_b\\ V_c\end{bmatrix} \quad \text{and} \quad \mathbf{I}_a = \begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}.$$

Owing to the varying exponential terms in the above operators, the voltage equation cannot be decomposed into linear differential equations with constant coefficients. However, from a study of the form of $\mathbf{Z}$ in Eq. 7, it is readily seen that unitary transformations with variable base vectors may lead to the linearization of the differential matrix equations.

UNITARY TRANSFORMATION WITH VARIABLE BASE VECTORS

It is interesting to note that the first three matrices occurring in Eq. 7 characterize the internal reaction of a completely symmetrical machine, while the remaining ones describe the feature of saliency. In fact, the main object of the above analysis is to consider the machine as a whole instead of treating its reactions as many reactance components from the beginning. From such a general impedance matrix, whose coefficients are expressible in terms of resistances, inductances, and the differential operators, it is at once apparent which terms taken by themselves give rise to complete symmetry, which terms, if coexistent with the former, will result in practical symmetry, and finally which terms in the latter are due to the saliency of poles.

Let us introduce the following unitary matrices:

$$\mathbf{U} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ \epsilon^{-j\theta} & a\epsilon^{-j\theta} & a^2\epsilon^{-j\theta} \\ \epsilon^{j\theta} & a^2\epsilon^{j\theta} & a\epsilon^{j\theta} \end{bmatrix} \tag{9}$$

and

$$\mathbf{U}^{-1} = \mathbf{U}_t^* = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \epsilon^{j\theta} & \epsilon^{-j\theta} \\ 1 & a^2\epsilon^{j\theta} & a\epsilon^{-j\theta} \\ 1 & a\epsilon^{j\theta} & a^2\epsilon^{-j\theta} \end{bmatrix} \tag{10}$$

and transform the voltage equation (8).

We get

$$\mathbf{U}\cdot(\mathbf{V}_a - \mathbf{E}_a) = \mathbf{U}\cdot\mathbf{Z}\cdot\mathbf{U}_t^*\cdot\mathbf{U}\cdot\mathbf{I}_a. \tag{11}$$

Let the components of the new voltage and current matrices be:

$$\mathbf{U}\cdot(\mathbf{V}_a - \mathbf{E}_a) = \begin{bmatrix} V_0 \\ V_s \\ V_n \end{bmatrix} \quad \text{and} \quad \mathbf{U}\cdot\mathbf{I}_a = \begin{bmatrix} i_0 \\ i_s \\ i_n \end{bmatrix}.$$

The transformed equation takes the following form:

$$\begin{bmatrix} V_0 \\ V_s \\ V_n \end{bmatrix} = \begin{bmatrix} \gamma_a+(L_{a0}-2M_{a0})p & \left(\frac{L_{a2}-M_{a2}}{2}\right)\epsilon^{j3\theta} & \left(\frac{L_{a2}-M_{a2}}{2}\right)\epsilon^{-j3\theta} \\ \left(\frac{L_{a2}-M_{a2}}{2}\right)(p+j\nu)\epsilon^{-j3\theta} & \gamma_a+L_a(p+j\nu)-3A(p+j\nu) & \frac{3}{2}L_{a2}(p+j\nu)-3B(p+j\nu) \\ \left(\frac{L_{a2}-M_{a2}}{2}\right)(p-j\nu)\epsilon^{j3\theta} & \frac{3}{2}L_{a2}(p-j\nu)-3B^*(p-j\nu) & \gamma_a+L_a(p-j\nu)-3A^*(p-j\nu) \end{bmatrix}\begin{bmatrix} i_0 \\ i_s \\ i_n \end{bmatrix}. \tag{12}$$

From Eq. 12, it may be noted that if the stator current of the machine is constrained to be strictly sinusoidal of fundamental frequency, there will be a third-harmonic voltage drop in the internal impedance due to the inequality of L_{a2} and M_{a2}. Conversely, if the line-to-neutral voltage is constrained to be positive sequence of fundamental voltage, a zero-sequence third-harmonic current will appear. However, if the operation does not involve neutral, there could be no zero-sequence current and both the voltage and the current are sinusoidal.

By assuming $L_{a2} = M_{a2}$, which is approximately true in practice, Eq. 12 simplifies to

$$\begin{bmatrix} V_0 \\ V_s \\ V_n \end{bmatrix} = \begin{bmatrix} \gamma_a+(L_{a0}-2M_{a0})p & 0 & 0 \\ 0 & \gamma_a+L_a(p+j\nu)-3A(p+j\nu) & \frac{3}{2}L_{a2}(p+j\nu)-3B(p+j\nu) \\ 0 & \frac{3}{2}L_{a2}(p-j\nu)-3B^*(p-j\nu) & \gamma_a+L_a(p-j\nu)-3A^*(p-j\nu) \end{bmatrix}\begin{bmatrix} i_0 \\ i_s \\ i_n \end{bmatrix}. \tag{13}$$

Equations 12 and 13 are linear differential equations, the operational solutions can be easily obtained and interpreted by Laplace or Heaviside method. We shall not, however, go into details in this paper.

REFERENCE FRAMES

The reference frames of machines have recently been discussed by several machine experts. Kron (2) has classified the reference frames of a synchronous machine. Ku (3) in his recent paper has emphasized the advantages of rotating axes for the transient analysis of both rotating and stationary networks. Rotating reference axes are necessary for balanced polyphase machines whose axes may be assumed rotating with the revolving field, thereby reducing the analysis to that of direct current machines. It is important in hunting studies. Park's (4) two reaction theory recently discussed by Adkins (5) and Vowels (6) employs rotating axes in the transient study of synchronous machines. It simplifies the problems by transforming the machine from slip ring to two quadrature brush sets, and the resultant direct and quadrature currents become direct currents.

In a previous paper (7) the author pointed out that the normalized Clarke's co-ordinates and Park's two-reaction co-ordinates may be obtained from the phase axes by orthogonal transformations, while the stationary and the rotating symmetrical co-ordinates are obtained by unitary transformations. In fact, the normalized Clarke's co-ordinates are related to the stationary symmetrical co-ordinates in the same way as the two-reaction co-ordinates are relate.. to the rotating symmetrical co-ordinates.

To correlate Eq. 13 with Park's result, we may introduce the following simple transformation (8):

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & \frac{j}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_s \\ V_n \end{bmatrix} \tag{14}$$

and transform (13) to

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} D_d(p) & D_q(p) \\ Q_d(p) & Q_q(p) \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{15}$$

where

$$D_d(p) = \gamma_a + (L_a + \tfrac{3}{2}L_{a2})p - \tfrac{3}{2}\Delta p^2$$

$$D_q(p) = -(L_a - \tfrac{3}{2}L_{a2})\nu + \tfrac{3}{2}\nabla\nu p$$

$$Q_d(p) = (L_a + \tfrac{3}{2}L_{a2})\nu - \tfrac{3}{2}\Delta\nu p$$

$$Q_q(p) = \gamma_a + (L_a - \tfrac{3}{2}L_{a2})p - \tfrac{3}{2}\nabla p^2$$

and

$$\Delta = \frac{M_{1a}^2(\gamma_2 + L_2 p) - 2M_{1a}M_{2a}M_{12}p + M_{2a}^2(\gamma_1 + L_1 p)}{(\gamma_1 + L_1 p)(\gamma_2 + L_2 p) - M_{12}^2 p^2}$$

$$\Delta = \frac{M_{3q}^2}{\gamma_3 + L_3 p}.$$

For steady state synchronous operation, $p = 0$, $\nu = \omega$, the above operational two reaction impedance functions reduce to

$$D_d(0) = \gamma_a, \qquad Q_q(0) = \gamma_a$$

$Q_d(0) = \omega(L_a + \frac{3}{2}L_{a2}) = \omega L_d = X_d$, the direct axis synchronous reactance

$D_q(0) = -\omega(L_a - \frac{3}{2}L_{a2}) = -\omega L_q = X_q$, the quadrature axis synchronous reactance.

PERFORMANCE CHARACTERISTIC ON BALANCED VOLTAGES

Equations 12 and 13 give all the information both transient and steady state of the machine under any terminal conditions. We shall investigate, however, the performance of a machine under balanced voltage. Let the voltage applied to the motor be a balanced positive sequence given by

$$\mathbf{V}_a = \sqrt{2}V \begin{bmatrix} \cos \omega t \\ \cos(\omega t - 120) \\ \cos(\omega t + 120) \end{bmatrix} \tag{16}$$

then

$$\mathbf{U} \cdot \mathbf{V}_a = \frac{V}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \epsilon^{-j\theta} & a\epsilon^{-j\theta} & a^2\epsilon^{j\theta} \\ \epsilon^{j\theta} & a^2\epsilon^{j\theta} & a\epsilon^{j\theta} \end{bmatrix} \begin{bmatrix} \cos \omega t \\ \cos(\omega t - 120) \\ \cos(\omega t + 120) \end{bmatrix} = \sqrt{\tfrac{3}{2}}V \begin{bmatrix} 0 \\ \epsilon^{js\omega t} \\ \epsilon^{-js\omega t} \end{bmatrix} \tag{17}$$

where s is the slip under asynchronous operations.

Substituting (17) into (13) and solving for i_s and i_n, we get

$$i_s = \frac{\sqrt{\frac{3}{2}}V\epsilon^{js\omega t}[\gamma_a + L_a(p+j\nu) - 3A(p+j\nu)] - \sqrt{\frac{3}{2}}V\epsilon^{-js\omega t}[L'_{a2}(p+j\nu) - 3B(p+j\nu)]}{D(p)} \tag{18}$$

$$i_n = \frac{\sqrt{\frac{3}{2}}V\epsilon^{-js\omega t}[\gamma_a + L_a(p-j\nu) - 3A^*(p-j\nu)] - \sqrt{\frac{3}{2}}V\epsilon^{-js\omega t}[L'_{a2}(p-j\nu) - 3B^*(p-j\nu)]}{D(p)} \tag{19}$$

wherein

$$L'_{a2} = \tfrac{3}{2}L_{a2}$$

and

$$D(p) = \begin{vmatrix} \gamma_a + L_a(p+j\nu) - 3A(p+j\nu) & L'_{a2}(p+j\nu) - 3B(p+j\nu) \\ L'_{a2}(p-j\nu) - 3B^*(p-j\nu) & \gamma_a + L_a(p+j\nu) - 3A^*(p-j\nu) \end{vmatrix}. \tag{20}$$

To obtain the steady-state value, it is only necessary to change p in the operators to $js\omega$ or $-js\omega$ as the case may be. Then we get

$$I_{s(s.s.)} = \frac{\sqrt{\frac{3}{2}}V\epsilon^{js\omega t}[\gamma_a + j(s\omega + \nu)L_a - 3Aj(s\omega + \nu)]}{D(js\omega)} - \frac{\sqrt{\frac{3}{2}}V\epsilon^{-js\omega t}[j(\nu - s\omega)L'_{a2} - 3Bj(\nu - s\omega)]}{D(-js\omega)} \quad (21)$$

$$I_{n(s.s.)} = \frac{\sqrt{\frac{3}{2}}V\epsilon^{-js\omega t}[\gamma_a - j(s\omega + \nu)L_a + 3A^*(s\omega + \nu)]}{D(-js\omega)} - \frac{\sqrt{\frac{3}{2}}V\epsilon^{js\omega t}[j(s\omega - \nu)L'_{a2} - 3B^*j(s\omega - \nu)]}{D(js\omega)}. \quad (22)$$

The above two expressions can be conveniently put into the following forms:

$$I_{s(s.s.)} = \sqrt{\frac{3}{2}}\frac{V}{\dot{Z}_1}\epsilon^{js\omega t} - \sqrt{\frac{3}{2}}\frac{V}{\dot{Z}_2}\epsilon^{-js\omega t} \quad (23)$$

$$I_{n(s.s.)} = \sqrt{\frac{3}{2}}\frac{V}{\dot{Z}_1^*}\epsilon^{-js\omega t} - \sqrt{\frac{3}{2}}\frac{V}{\dot{Z}_2^*}\epsilon^{js\omega t}. \quad (24)$$

If the operation does not involve neutral, the line current can be calculated from

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \epsilon^{j\theta} & \epsilon^{-j\theta} \\ 1 & a^2\epsilon^{j\theta} & a\epsilon^{-j\theta} \\ 1 & a\epsilon^{j\theta} & a^2\epsilon^{-j\theta} \end{bmatrix}\begin{bmatrix} 0 \\ i_s \\ i_n \end{bmatrix}. \quad (25)$$

Hence the current in line a is of the form,

$$i_a = I_\omega \cos(\omega t + \phi_1) + I_{(2s-1)\omega}\cos[(2s - 1)\omega t + \phi_2] \quad (26)$$

where

$$I_\omega = \frac{\sqrt{2}V}{\sqrt{\dot{Z}_1\dot{Z}_1^*}} \qquad I_{(2s-1)\omega} = \frac{\sqrt{2}V}{\sqrt{\dot{Z}_2\dot{Z}_2^*}}. \quad (27)$$

Equation 26 shows that the stator currents consist of two frequencies and the current of each frequency forms a balanced system.

The average torque may be calculated as the algebraic sum of the torques produced by each balanced system as if the other were absent. For rotor speed between standstill and half synchronous value, the torque of currents having $(2s - 1)$ times impressed frequency is negative and should be subtracted from that produced by currents of impressed frequency. The exact shape of the resultant curve depends on the constants of the machine. However, the well-known Georges

phenomenon (**9**) of having a dip of the resultant torque near half-synchronous speed is present in every case.

SPECIAL CASES

Let us consider the simpler case of machines having main windings only. The operators in Eq. 13 reduce to the following:

$$A(p+j\nu) = \frac{M^2p(p+j\nu)}{4(\gamma_1+L_1p)} \qquad A^*(p-j\nu) = \frac{M^2p(p-j\nu)}{4(\gamma_1+L_1p)}$$

$$A(p+j\nu) = B(p+j\nu) \qquad A^*(p-j\nu) = B^*(p-j\nu) \tag{28}$$

and the denominator becomes

$$\begin{aligned} D(p) = {} & [\gamma_a + L_a(p+j\nu)][\gamma_a + L_a(p-j\nu)](\gamma_1+L_1p) \\ & - [\gamma_a + L_a(p-j\nu)]\tfrac{3}{4}M^2p(p+j\nu) \\ & - [\gamma_a + L_a(p+j\nu)]\tfrac{3}{4}M^2p(p-j\nu). \end{aligned} \tag{29}$$

Therefore, the impedances, $\dot{Z}_1$ and $\dot{Z}_2$, in Eqs. 23 and 24 are given below:

$$\dot{Z}_1 = \frac{(\gamma_a+jX_a)[\gamma_a+j(2s-1)X_a](\gamma_1+jsX_1)+\frac{3}{2}X_m^2s[s\gamma_a+j(2s-1)(X_a-X_{2a})]+(2s-1)X_{2a}^2(\gamma_1+jsX_1)}{[\gamma_a+j(2s-1)X_a]\left[\frac{\gamma_1}{s}+jsX_1\right]+\frac{3}{4}X_m^2s(2s-1)} \tag{30}$$

$$\dot{Z}_2 = \frac{(\gamma_a-jX_a)[\gamma_a+j(2s-1)X_a](\gamma_1-jsX_1)+(2s-1)X_{2a}^2(\gamma_1+jsX_1)+\frac{3}{2}sX_{1n}^2[s\gamma_a-j(2s-1)(X_a-X_{2a})]}{j(\gamma_1-jsX_1)(2s-1)X_{2a}-\frac{4}{3}X_m^2s(2s-1)} \tag{31}$$

(*a*) *Synchronous Speed*, $s = 0$

For synchronous operation,

$$\dot{Z}_1 = \frac{\gamma_a^2 + X_a^2 - X_{2a}^2}{\gamma_a - jX_a} = \frac{\gamma_a^2 + X_dX_q}{\gamma_a - jX_a} \tag{32}$$

$$\dot{Z}_2 = \frac{\gamma_a^2 + X_a^2 - X_{2a}^2}{jX_{2a}} = \frac{\gamma_a^2 + X_dX_q}{jX_{2a}} \tag{33}$$

where $X_d = X_a + X_{2a}$ is the direct-axis synchronous reactance, and $X_q = X_a - X_{2a}$ is the quadrature-axis synchronous reactance.

The two components of stator current are of the same frequency; hence the two corresponding admittances, $\dot{Y}_1$ and $\dot{Y}_2$, may be added in series if the displacement angle is constrained to be zero. Thus, the impedance per phase is

$$\dot{Z} = \frac{1}{\dot{Y}_1 + \dot{Y}_2} = \frac{\gamma_a^2 + X_dX_q}{\gamma_a - jX_q} \cong \frac{X_d}{X_q}\gamma_a + jX_d \tag{34}$$

for γ_a^2 is smaller than either X_q^2 or X_dX_q.

Similarly with angle constrained to be 90°, the impedance per phase is

$$\dot{Z} = \frac{1}{\dot{Y}_1 - \dot{Y}_2} = \frac{\gamma_a^2 + X_d X_q}{\gamma_a - jX_d} \cong \frac{X_q}{X_d}\gamma_a + jX_q. \tag{35}$$

The above reveals, incidentally, the principle of the well known slip tests. Thus, if the machine is run at small slip, the ratio of the maximum voltage to the minimum current gives the direct-axis synchronous reactance, Eq. 34, and that of the minimum voltage to the maximum current, Eq. 35, gives the quadrature-axis synchronous reactance. For round-rotor machines, $L_{a2} = 0$, $\dot{Z}_2 = \infty$, the impedance per phase will have the value, $\dot{Z}_1 = \gamma_a + jX_a$, where $X_a = X_d = X_q$ is the synchronous reactance independent of the displacement angle.

(*b*) *Half-Synchronous Speed*, $s = \frac{1}{2}$

At half-synchronous speed, the current is of impressed frequency only. The internal impedance of the machine becomes simply

$$\dot{Z}_1 = \gamma_a + jX_a + \frac{\frac{3}{4}X_m^2}{2\gamma_1 + jX_1} \tag{36}$$

which does not depend upon the saliency of the machine and therefore is of the same form as that of a polyphase induction motor operated at half speed; however, in the latter case, the rotor reaction is

$$\frac{\frac{9}{4}X_m^2}{2\gamma_1 + jX_1}$$

instead, being three times that of the present case.

Assuming that the rotor resistance is negligible, Eq. 36 can be conveniently written as

$$\dot{Z}_1 = \gamma_a + jX_a\left(1 - \frac{\frac{3}{4}X_m^2}{X_a X_1}\right) = \gamma_a + j\tau X_a \tag{37}$$

where $\tau = 1 - \frac{\frac{3}{4}X_m^2}{X_a X_1}$ may be interpreted as the equivalent coefficient of coupling between the stator and the rotor.

The conductance and susceptance parts are

$$g = \frac{\gamma_a}{\gamma_a^2 + \tau^2 X_a^2} \quad \text{and} \quad b = \frac{\tau X_a}{\gamma_a^2 + \tau^2 X_a^2},$$

respectively.

(*c*) *Standstill, s = 1*

At standstill, the current is again of impressed frequency and the two admittances, $\dot{Y}_1$ and $\dot{Y}_2$, may be considered to be in parallel; their equivalent value is simply $\dot{Y}_1 + \dot{Y}_2$.

(*d*) *Infinite Speed, s = ∞*

The second term of Eq. 26 is of infinite frequency. Since infinite slip cannot be obtained physically, this is, of course, fictitious. It is interesting, however, to investigate the forms of $\dot{Z}_1$ and $\dot{Z}_2$ for these limiting conditions. In order to show the relationship of $\dot{Z}_1$ and $\dot{Z}_2$ with their values at synchronous or half-synchronous speeds, let us introduce the coefficient of coupling, σ, defined by

$$\sigma = 1 - \frac{3}{2}\frac{X_m^2}{X_a X_1}$$

and consider the simple case of cylindrical rotor only. Neglecting X_{2a}, we find

$$\dot{Z}_1 = \sqrt{\gamma_a^2 + X_a^2\left(\frac{\sigma}{\tau}\right)^2} \tag{38}$$

and

$$\dot{Z}_2 = \sqrt{\frac{\gamma_a^2 + X_a^2\left(\frac{\sigma}{\tau}\right)^2}{\left(\frac{\tau - 1}{\sigma}\right)^2}}. \tag{39}$$

Since the stator current in this case is not a simple sinusoidal wave of fundamental frequency, the equivalent admittance for infinite slip should be obtained by combining the two terms in the usual manner. Thus,

$$Y = \sqrt{\frac{1^2 + \left(\frac{\tau - 1}{\sigma}\right)^2}{\gamma_a^2 + X_a^2\left(\frac{\sigma}{\tau}\right)^2}}. \tag{40}$$

The ratio of power to the square of the voltage will give the conductance part of the admittance. Hence,

$$g = \frac{\gamma_a}{\gamma_a^2 + X_a^2\left(\frac{\sigma}{\tau}\right)^2} \qquad \text{and} \qquad b = \sqrt{Y^2 - g^2}.$$

The value of the susceptance will be approximately the same as Y since g is usually small compared with b.

CONCLUSION

It can readily be inferred from the preceding treatment that the unitary transformation of Eq. 18 is just as useful and usable in studying the performance of partly symmetrical machines as Park's transformation. Moreover, it can throw additional light on the subject when Park's transformation fails to show. For example, under steady state balanced operation, Eq. 13 becomes simply

$$(\gamma_a + jX_a)\dot{I} - jX_{a2}\dot{I}^* = \dot{E} - \dot{V} \qquad (13a)$$

where $X_a = \omega L_a$, $X_{a2} = \omega L'_{a2}$, $\dot{I}$ is the armature current, $\dot{I}^*$ its conjugate. If $\dot{E}$ is taken as the axis of reference, and the angle between $\dot{E}$ and $\dot{I}$ be α, (13*a*) becomes

$$[(\gamma_a + X_{a2}\sin 2\alpha) + j(X_a - X_{a2}\cos 2\alpha)]\dot{I} = \dot{E} - \dot{V}. \qquad (13b)$$

Note that the term $X_{a2}\sin 2\alpha$ corresponds to an additional resistance, and when $\sin 2\alpha$ is negative, we have the effect of negative resistance. This seems to be a novel feature of the saliency effect.

Acknowledgment

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APPENDIX

Rotor Reaction

The rotor reaction of the machine is $p\mathbf{M}_{af}\cdot\mathbf{Y}_{ff}\cdot p\mathbf{M}_{fa}$. Since $\mathbf{M}_{af}$ is the transpose of $\mathbf{M}_{fa}$, hence by Eqs. 4 and 5, after carrying out the multiplication, we get the four matrices given in Eq. 6. The operational functions, $A(p)$ and $B(p)$ are

$$A(p)=\tfrac{3}{2}p(p+j\nu)\left[\frac{M_{1a}^2(\gamma_2+L_2p)-2M_{1a}M_{2a}M_{12}p+M_{2a}^2(\gamma_1+L_1p)}{(\gamma_1+L_1p)(\gamma_2+L_2p)-M_{12}^2p^2}+\frac{M_{3a}^2}{\gamma_3+L_3p}\right]_{p\to(p-j\nu)} \tag{41}$$

$$B(p)=\tfrac{3}{2}p(p+j\nu)\left[\frac{M_{1a}^2(\gamma_2+L_2p)-2M_{1a}M_{2a}M_{12}p+M_{2a}^2(\gamma_1+L_1p)}{(\gamma_1+L_1p)(\gamma_2+L_2p)-M_{12}^2p^2}-\frac{M_{3a}^2}{\gamma_3+L_3p}\right]_{p\to(p-j\nu)} \tag{42}$$

where the symbol, $p\to(p-j\nu)$, means that all p's on the right of the above equation are to be replaced by $p-j\nu$. $A^*(p)$ and $B^*(p)$ are the corresponding conjugate functions.

Automatic Monitor for Radiochemical Labs.—A versatile new "hand and foot counter" for automatic monitoring of beta-gamma contamination on the hands and feet of radioisotope technicians has been announced by Nuclear Instrument & Chemical Corp., Chicago. The new model monitor is a five channel scaler and power supply system with automatic controls to time the counting operations in proper sequence. One scaling channel is provided for each side of each hand, and one for the soles of both feet.

Typical operation finds a green "ready to use" sign lighted when the user steps up to be tested. The counting process is started and maintained by pressure of the fingertips on a flexible plate at the rear of the hand pockets. This lights up a white "counter in operation" sign. During the test, the user sees five banks of decade indicators at eye level in front of him. Each of these decades will present a single number to indicate the degree of surface contamination. After a preset time, the timer will shut off the scalers. A blue sign will indicate "check OK," or a red sign will show "decontamination required." If the hands are removed or the counting operation interrupted before the complete time has elapsed, yellow "check incomplete—reset and repeat" and "push to reset" legends light. If the number of counts in any channel exceeds the scaling factor, the decontamination light goes on at the end of the counting period.

A button is available for manual resetting. Ordinarily, the unit automatically resets itself after a predetermined time. Intervals are variable from six to sixty seconds.

All controls are locked inside to prevent tampering and accidents. The unit is grounded to prevent the user from contacting any potential above ground.

Scintillation Counter for Asphalt.—A General Electric scintillation counter is an important part of a new test method aimed at producing better highways with the aid of atomic radiation.

In the laboratories of The Atlantic Refining Company, Philadelphia, manufacturers of paving asphalt, typical road stone is soaked in radioactive calcium chloride and dried. It is then coated evenly with asphalt, which may contain additives to improve the adhesive characteristics, and the mixture is covered with water for a specified period of time.

If the asphalt separates from the stones, the radioactive calcium chloride dissolves into the water. After evaporation, the radioactivity of the residue is measured with the G-E scintillation counter. A correlation between the radioactivity count and previously prepared charts tells the Atlantic engineers the quality and adhesive characteristics of the asphalt.

The G-E scintillation counter is designed to detect alpha, beta, gamma, and thermal neutron radiation. This is done by placing various phosphors inside the "chimney" of the equipment. As radiation strikes the phosphor, it excites it so that it gives off a certain amount of "twinkling" (scintillation), intensity of which varies with the amount of radiation. By counting this scintillation, the instrument accurately measures the radiation intensity.

UNBALANCES OF UNTRANSPOSED OVERHEAD LINES *

BY

ERIC T. B. GROSS [1]

SYNOPSIS

Improvements in equipment and new developments in both the communication and power fields have greatly reduced the need for transpositions at close intervals on overhead high voltage lines. In many systems transpositions are now made only at switchyards and are kept to a minimum.

Geometric unbalance of untransposed high voltage lines leads to circulating residual ground currents in systems which are solidly grounded, to neutral voltages to ground in impedance and resonant grounded systems, and to negative sequence currents which flow also in transformers and generators. Experience has been very satisfactory on systems in which line transpositions were eliminated; however, it is desirable to know the magnitudes of the unbalanced currents and voltages. Practical methods for the computation of these residual and negative sequence quantities are summarized in this paper.

INTRODUCTION

Not long ago, power-transmission engineers felt that transpositions at fairly close intervals were a necessity on high voltage lines in order to coordinate communication and power systems. However, in several high voltage systems in this country, the number of transposition structures has been greatly reduced and the result has been very satisfactory operating experience. Many new lines have been built with no transpositions and very few remedial measures toward inductive coordination became necessary after these lines were energized and placed in service. Various components contributed to the change in practice regarding transposition structures. Separate rights of way for communication and power lines, use of underground cables or otherwise shielded circuits for communication purposes, reduced susceptibility to noise of the communication equipment, better wave shapes in improved design of transformers and rotating machines are some of the reasons which have greatly influenced our thinking relative to the need for transpositions.

Another very important reason for the trend toward reducing the number of transpositions on line sections is based on the experience that on some systems not less than one out of four faults was physically associated with a transposition. Of course, elimination of transpositions and the necessary structures produces savings which may also be very worthwhile.

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UNBALANCED CURRENTS AND VOLTAGES

It is difficult, if not impossible, to arrange the conductors of a three-phase transmission line on the towers in such a manner that both the electric and magnetic fields, produced by the charges and currents respectively, are completely symmetrical. Complete transposition results in a high degree of balance between the sending and receiving ends of lines or line-sections so transposed. Omitting transpositions between stations will therefore enlarge the effect of the geometric unbalance of the conductor configuration. As a result of electrostatic or electromagnetic unbalance circulating residual currents will flow in high voltage

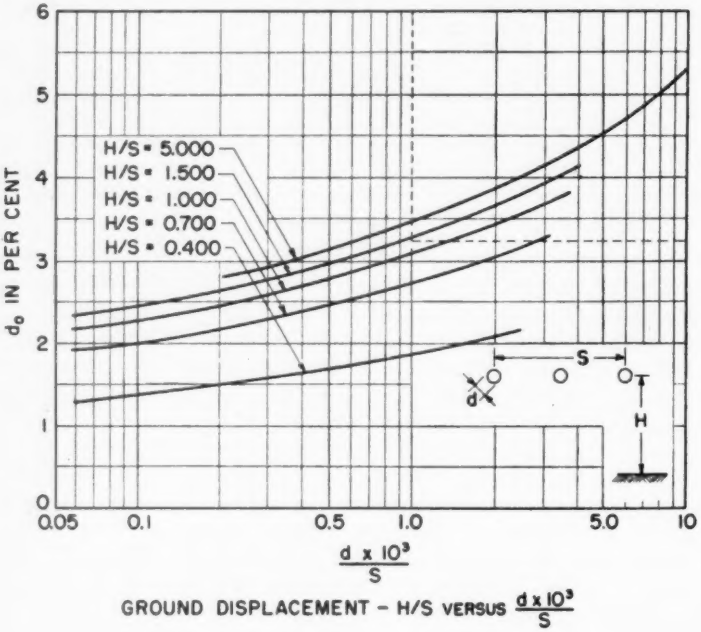

FIG. 1.

systems with solidly grounded neutrals. In high impedance or in resonant grounded systems voltages between system neutral and ground (neutral displacements) will be produced. However, the unbalance is not restricted to residual or zero-sequence quantities; negative sequence charging currents will flow in the line because of capacitive unbalance, and magnetically induced negative sequence currents will flow through the lines and also the windings of transformers and rotating machines. The residual currents may affect the proper operation of very sensitive elements in ground relays, and negative sequence currents may lead to undesirable additional losses in rotating machines.

The experience gained by the operating engineers has nowhere lead to a critical situation. This indicates that the unbalances are small and not very important. However, it is well to know beforehand what the magnitude of the unbalanced quantities may be. The basic relations have been established some time ago (1, 2) [2] and are available in the references. However some extension (3) seemed desirable to include the effect of ground wires on the residual ground current (that is, the charging currents to ground).

The residual current $I_u = 3I'_{a0}$ may be evaluated from the "ground displacement" d_0, a quantity which can be computed when the physical data of the line in consideration are known (3). What is needed are the conductor and groundwire sizes, their spacings and their average heights above ground, that is, their height at the supports minus two-thirds of the sag. The steps required for the evaluation of d_0 in a specific case may be taken from the general analysis contained in Appendix 1. Of considerable practical value should be Fig. 1, in which is drawn a family of curves which gives directly the magnitude of d_0 for any special case of a line with horizontal conductor configuration and without groundwires. These curves cover conductor sizes from the very small wire sizes #16 (d = 0.0508-in. diameter) to 1590 MCM–ACSR (d = 1.545-in. diameter), average conductor heights H above ground from 25 ft. to 60 ft., and spacings S between the two outer conductors from 12 ft. to 64 ft. The variables in Fig. 1 are ratios d/S and H/S; it may be noted that all quantities in this ratio must be expressed in the same units, that is, inches must be changed to feet, or vice-versa.

The higher the voltage, the larger will be S; therefore, the lowest of the curves, for a small H/S ratio, refers to lines operating at higher voltages and it is seen that the unbalance d_0 becomes smaller with increasing voltage. The dotted lines in the upper right corner of the figure indicate the region which is covered by somewhat similar curves in Ferris' report (1). When his investigation was made in 1916, the high voltages of today were but a dream. Similar curves for lines with ground wires and for other configurations are in preparation; they require a considerable amount of computation.

The magnitudes of zero-sequence charging currents I'_{a0} and of negative-sequence charging currents I'_{a2} may be computed by the methods of Appendix 2. Certain approximations make the results somewhat less accurate than those of the analysis outlined in Appendix 1, but what is important in such an analysis is the order of magnitude rather than very accurate values.

The magnitudes of zero-sequence currents I_{a0} and of negative-sequence currents I_{a2} induced in the line electromagnetically, because of geometric unbalance, by the load current I_{a1} may be found from the equations (2) given in Appendix 3.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The general approach to this problem is basically the same for more complex configurations, such as double circuit lines or twin conductor (Buendle conductor) lines, with and without groundwires.

Since the charging currents and load currents are essentially independent of each other, their unbalance components (I_u, I'_{a2}, and I_{a0}, I_{a2}) may be evaluated separately and superposed without regard to a difference in phase angle in order to give an indication of the worst, but unlikely condition.

CONCLUSION

1. It is desirable to compute unbalanced currents of line sections without transpositions.

2. The ground displacement d_0 (Fig. 1) enables a quick and accurate determination of residual ground currents (Appendix 1).

3. Symmetrical component analysis of untransposed lines leads to simple and practical formulae for the evaluation of approximate negative and zero sequence charging and load currents (Appendix 2 and Appendix 3).

4. These circulating currents may be reduced by phase interchanges or by complete transpositions at switchyards and stations.

5. Overhead lines without transpositions have been in operation on many systems for long periods without any noticeable detrimental effects.

6. Elimination of transposition towers prevents troubles on such towers which seem to be a repeated cause for disturbances.

Acknowledgment

I am indebted to Mr. Cecil T. Wint for his help in carrying out the computations on which Fig. 1 is based.

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APPENDIX 1

GROUND DISPLACEMENT OF UNTRANSPOSED LINES

The relations between the charges (**Q**) on the conductors 1, 2, and 3 and their voltages to ground (**V**) are expressed by the following equations:

$$\begin{aligned} \mathbf{V}_1 &= P_{11}\mathbf{Q}_1 + P_{12}\mathbf{Q}_2 + P_{13}\mathbf{Q}_3 \\ \mathbf{V}_2 &= P_{12}\mathbf{Q}_1 + P_{22}\mathbf{Q}_2 + P_{23}\mathbf{Q}_3 \\ \mathbf{V}_3 &= P_{13}\mathbf{Q}_1 + P_{23}\mathbf{Q}_2 + P_{33}\mathbf{Q}_3. \end{aligned} \qquad (A)$$

The potential coefficients P_{ik} are given by the expressions:

$$P_{ii} = R \log \frac{D_{ii}}{r_{ii}} \quad \text{and} \quad P_{ik} = R \log \frac{D_{ik}}{d_{ik}} \quad (i \neq k). \qquad (B)$$

D, d and r are measured in the same units. Figure 2 indicates what these letters stand for.

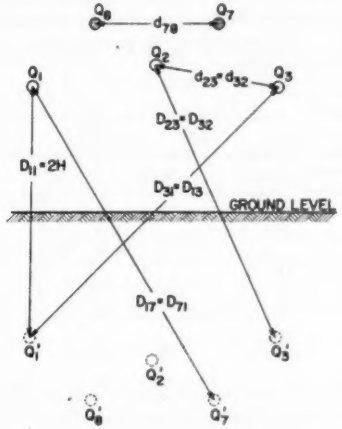

FIG. 2. Image charges and separations.

For our purpose, the factor of proportionality (R) may be made equal to an arbitrary value, since only ratios of P's appear in the pertinent end result. The numerical values of the self-potential coefficients P_{ii} are about one decimal order larger than those of the mutual potential coefficients P_{ik} $(i \neq k)$. Values for a typical, modern high-voltage transmission line are $P_{ii} = 3.4$ and $P_{ik} = 0.7$ (R assumed equal to one).

The ground displacement (**3**) is defined as $d_0 = \dfrac{E_0}{E_{ph}}$. In this equation, E_{ph} is the phase to neutral voltage of a three phase system of impressed balanced line-to-line voltages, and E_0 is the voltage from its system neutral to ground, that is, the zero sequence voltage of the three line-to-ground voltages in the ungrounded system. Evidently, E_0 depends only on the capacitances to ground C_{ag}, C_{bg}, and C_{cg} (Fig. 3), and not on the capacitances between the three phases C_{ab}, C_{ac}, and C_{bc}.

For the specified conditions, and by using the complex operator **a**, we get:

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{E}_1 + \mathbf{E}_0 \\ \mathbf{V}_2 &= \mathbf{E}_2 + \mathbf{E}_0 = \mathbf{a}^2\mathbf{E}_1 + \mathbf{E}_0 \\ \mathbf{V}_3 &= \mathbf{E}_3 + \mathbf{E}_0 = \mathbf{a}\mathbf{E}_1 + \mathbf{E}_0. \end{aligned} \qquad (C)$$

Evidently, the sum of the charging currents to ground (Fig. 3) must be equal to zero, that is, $\mathbf{I}_{ag} + \mathbf{I}_{bg} + \mathbf{I}_{cg} = 0$, since they have no return path to the system. The sum of the charging currents $\mathbf{I}_{ab} + \mathbf{I}_{bc} + \mathbf{I}_{ca}$ must also be zero. Therefore, the charges $\mathbf{Q}$ on the conductors must add up to zero, so that four equations result from these relations and from (*A*), namely:

$$\begin{aligned} \mathbf{E}_1 + \mathbf{E}_0 &= P_{11}\mathbf{Q}_1 + P_{12}\mathbf{Q}_2 + P_{13}\mathbf{Q}_3 \\ \mathbf{a}^2\mathbf{E}_1 + \mathbf{E}_0 &= P_{12}\mathbf{Q}_1 + P_{22}\mathbf{Q}_2 + P_{23}\mathbf{Q}_3 \\ \mathbf{a}\mathbf{E}_1 + \mathbf{E}_0 &= P_{13}\mathbf{Q}_1 + P_{23}\mathbf{Q}_2 + P_{33}\mathbf{Q}_3 \\ 0 &= \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3. \end{aligned} \qquad (D)$$

Since there are four unknowns ($\mathbf{E}_0$, $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$) in these four equations, they can be solved for $\frac{\mathbf{E}_0}{\mathbf{E}_1}$ and for $d_0 = \frac{E_0}{E_1}$ in terms of the known P_{ik} if the arbitrary factor is made $R = \mathbf{E}_1$.

The results of computations for single circuit, three-phase lines without ground wires are summarized in Fig. 1. This family of curves covers the complete range of present day designs with horizontal arrangement of phase conductors. H is the average height above ground which includes the influence of sag. All dimensions are to be expressed in feet, the conductor outside diameter d must also be expressed in feet.

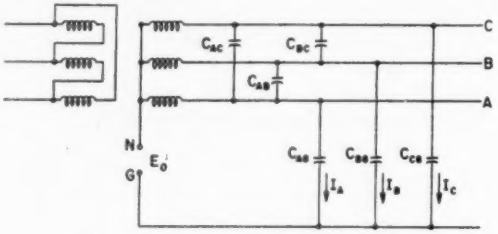

FIG. 3. Circuit diagram.

Three equations of the same form as (*A*) may be derived (Eqs. (*A*) of Appendix 2) for more general configurations such as lines with ground wires. Double circuit lines and twin conductor lines may be dealt with in a similar manner.

In solidly grounded systems, the neutral displacement will be zero, $\mathbf{E}_0 = 0$, but a residual or unbalanced current $\mathbf{I}_u = 3\mathbf{I}'_{a0}$ will flow in the ground connection of the transformer neutral. $\mathbf{I}'_{a0}$ is the zero-sequence component of the line charging currents. The ground displacement is also (3) equal to

$$d_0 = \frac{I_u}{I_G}.$$

I_G is the ground fault current of the ungrounded system (Fig. 3) during a single line to ground fault. The ratio d_0 may be computed, or obtained from Fig. 1, and I_G may be calculated from the zero-sequence capacitive reactance $\mathbf{X}'_0$ and the phase voltage $\mathbf{E}_1$. Then $\mathbf{I}_u = 3\mathbf{I}'_{a0} = \mathbf{d}_0\mathbf{I}_G$.

APPENDIX 2

SEQUENCE CHARGES ON UNTRANSPOSED LINES

If the three phases are called a, b, c, and the ground wires w and x, the following equations[3] relate the charges $\mathbf{Q}$, voltages to ground $\mathbf{E}$ and potential coefficients P'_{ik}:

[3] In this appendix, the P_{ik} of Appendix 1 are called P'_{ik}. In general, the phase wires are numbered 1, 2, 3, 4, 5, 6 and the ground-wires 7 and 8. In this example of a single circuit line, the phase wires are identified by a, b, c, the ground-wires by w and y, (see also Fig. 3), in order to avoid confusion with subscripts a, 1, 2, 0 indicating symmetrical component quantities.

$$\begin{aligned}
\mathbf{E}_a &= P'_{aa}\mathbf{Q}_a + P'_{ab}\mathbf{Q}_b + P'_{ac}\mathbf{Q}_c + P'_{aw}\mathbf{Q}_w + P'_{az}\mathbf{Q}_z \\
\mathbf{E}_b &= P'_{ab}\mathbf{Q}_a + P'_{bb}\mathbf{Q}_b + P'_{bc}\mathbf{Q}_c + P'_{bw}\mathbf{Q}_w + P'_{bz}\mathbf{Q}_z \\
\mathbf{E}_c &= P'_{ac}\mathbf{Q}_a + P'_{bc}\mathbf{Q}_b + P'_{cc}\mathbf{Q}_c + P'_{cw}\mathbf{Q}_w + P'_{cz}\mathbf{Q}_z \\
0 &= P'_{aw}\mathbf{Q}_a + P'_{bw}\mathbf{Q}_b + P'_{cw}\mathbf{Q}_c + P'_{ww}\mathbf{Q}_w + P'_{wz}\mathbf{Q}_z \\
0 &= P'_{az}\mathbf{Q}_a + P'_{bz}\mathbf{Q}_b + P'_{cz}\mathbf{Q}_c + P'_{wz}\mathbf{Q}_w + P'_{zz}\mathbf{Q}_z.
\end{aligned}$$

Computing $\mathbf{Q}_w$ and $\mathbf{Q}_z$ from the last two equations and substituting their values into the first three equations enables elimination of the ground wire equations. These substitutions lead to new equations for the three voltages:

$$\begin{aligned}
\mathbf{E}_a &= P_{aa}\mathbf{Q}_a + P_{ab}\mathbf{Q}_b + P_{ac}\mathbf{Q}_c \\
\mathbf{E}_b &= P_{ab}\mathbf{Q}_a + P_{bb}\mathbf{Q}_b + P_{bc}\mathbf{Q}_c \\
\mathbf{E}_c &= P_{ac}\mathbf{Q}_a + P_{bc}\mathbf{Q}_b + P_{cc}\mathbf{Q}_c.
\end{aligned} \qquad (A)$$

Replacing the P_{ik} by self and mutual sequence-potential coefficients $\mathbf{P}_{mn}$ ($m, n = 1, 2, 0$), in line with a similar [4] analysis made by Edith Clarke, leads to the equations:

$$\begin{aligned}
\mathbf{E}_{a1} &= \mathbf{P}_{11}\mathbf{Q}_{a1} + \mathbf{P}_{12}\mathbf{Q}_{a2} + \mathbf{P}_{10}\mathbf{Q}_{a0} \\
\mathbf{E}_{a2} &= \mathbf{P}_{21}\mathbf{Q}_{a1} + \mathbf{P}_{22}\mathbf{Q}_{a2} + \mathbf{P}_{20}\mathbf{Q}_{a0} \\
\mathbf{E}_{a0} &= \mathbf{P}_{01}\mathbf{Q}_{a1} + \mathbf{P}_{02}\mathbf{Q}_{a2} + \mathbf{P}_{00}\mathbf{Q}_{a0}, \quad \text{where}
\end{aligned} \qquad (B)$$

α) for a Delta-system: $\mathbf{E}_{a0} \neq 0$ and $\mathbf{Q}_{a0} = 0$, since $\mathbf{Q}_a + \mathbf{Q}_b + \mathbf{Q}_c = 0$;

β) for a grounded system: $\mathbf{E}_{a0} = 0$ and $\mathbf{Q}_{a0} \neq 0$, since $\mathbf{Q}_a + \mathbf{Q}_b + \mathbf{Q}_c = \mathbf{Q}_u = 3\mathbf{Q}_{a0}$, the unbalanced (residual) charge.

In order to find the $\mathbf{P}_{mn}$, we simply proceed in line with the definition of the sequence potential coefficients. The first subscript indicates the sequence voltage which is produced by the sequence charge, indicated by the second subscript. Therefore, if only positive sequence charges are present, we use the $\mathbf{P}_{m1}$ and the following voltages result:

$$\mathbf{E}_{a1} = \mathbf{P}_{11}\mathbf{Q}_{a1}, \qquad \mathbf{E}_{a2} = \mathbf{P}_{21}\mathbf{Q}_{a1} \qquad \text{and} \qquad \mathbf{E}_{a0} = \mathbf{P}_{01}\mathbf{Q}_{a1}. \qquad (C)$$

The total voltages produced by positive sequence charges $\mathbf{Q}_{a1}$, $\mathbf{Q}_{b1} = \mathbf{a}^2\mathbf{Q}_{a1}$ and $\mathbf{Q}_{c1} = \mathbf{a}\mathbf{Q}_{a1}$ are found from Eqs. A as:

$$\begin{aligned}
\mathbf{E}_a &= (P_{aa} + \mathbf{a}^2P_{ab} + \mathbf{a}P_{ac})\mathbf{Q}_{a1} \\
\mathbf{E}_b &= (P_{ab} + \mathbf{a}^2P_{bb} + \mathbf{a}P_{bc})\mathbf{Q}_{a1} \\
\mathbf{E}_c &= (P_{ac} + \mathbf{a}^2P_{bc} + \mathbf{a}P_{cc})\mathbf{Q}_{a1}.
\end{aligned}$$

Since

$$\begin{aligned}
\mathbf{E}_{a1} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{a}\mathbf{E}_b + \mathbf{a}^2\mathbf{E}_c) \\
&= \tfrac{1}{3}\mathbf{Q}_{a1}[P_{aa} + \mathbf{a}^2P_{ab} + \mathbf{a}P_{ac} + \mathbf{a}P_{ab} + P_{bb} + \mathbf{a}^2P_{bc} + \mathbf{a}^2P_{ac} + \mathbf{a}P_{bc} + P_{cc}] \\
&= \tfrac{1}{3}\mathbf{Q}_{a1}[(P_{aa} + P_{bb} + P_{cc}) - (P_{ab} + P_{ac} + P_{bc})], \text{ we find from } (C): \\
\mathbf{P}_{11} &= \tfrac{1}{3}(P_{aa} + P_{bb} + P_{cc}) - \tfrac{1}{3}(P_{ab} + P_{ac} + P_{bc}).
\end{aligned}$$

Similarly, from

$$\begin{aligned}
\mathbf{E}_{a2} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{a}^2\mathbf{E}_b + \mathbf{a}\mathbf{E}_c) \text{ follows} \\
\mathbf{P}_{21} &= \tfrac{1}{3}(P_{aa} + \mathbf{a}P_{bb} + \mathbf{a}^2P_{cc}) + \tfrac{2}{3}(\mathbf{a}^2P_{ab} + \mathbf{a}P_{ac} + P_{bc})
\end{aligned}$$

and from

$$\begin{aligned}
\mathbf{E}_{a0} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{E}_b + \mathbf{E}_c) \text{ follows} \\
\mathbf{P}_{01} &= \tfrac{1}{3}(P_{aa} + \mathbf{a}^2P_{bb} + \mathbf{a}P_{cc}) - \tfrac{1}{3}(\mathbf{a}P_{ab} + \mathbf{a}^2P_{ac} + P_{bc}).
\end{aligned}$$

If only negative sequence charges are present, by definition of the $\mathbf{P}_{m2}$ follows:

$$\mathbf{E}_{a1} = \mathbf{P}_{12}\mathbf{Q}_{a2}, \qquad \mathbf{E}_{a2} = \mathbf{P}_{22}\mathbf{Q}_{a2} \qquad \text{and} \qquad \mathbf{E}_{a0} = \mathbf{P}_{02}\mathbf{Q}_{a2} \qquad (D)$$

[4] See Ref. 2, pp. 244–248 and p. 455.

therefore, from Eqs. *A*, since $\mathbf{Q}_{b2} = \mathbf{a}\mathbf{Q}_{a2}$ and $\mathbf{Q}_{c2} = \mathbf{a}^2\mathbf{Q}_{a2}$:

$$\mathbf{E}_a = (P_{aa} + \mathbf{a}P_{ab} + \mathbf{a}^2P_{ac})\mathbf{Q}_{a2}$$
$$\mathbf{E}_b = (P_{ab} + \mathbf{a}P_{bb} + \mathbf{a}^2P_{bc})\mathbf{Q}_{a2}$$
$$\mathbf{E}_c = (P_{ac} + \mathbf{a}P_{bc} + \mathbf{a}^2P_{cc})\mathbf{Q}_{a2}.$$

Since

$$\begin{aligned}\mathbf{E}_{a1} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{a}\mathbf{E}_b + \mathbf{a}^2\mathbf{E}_c)\\ &= \tfrac{1}{3}\mathbf{Q}_{a2}[P_{aa} + \mathbf{a}P_{ab} + \mathbf{a}^2P_{ac} + \mathbf{a}P_{ab} + \mathbf{a}^2P_{bb} + P_{bc} + \mathbf{a}^2P_{ac} + P_{bc} + \mathbf{a}P_{cc}]\\ &= \tfrac{1}{3}\mathbf{Q}_{a2}[(P_{aa} + \mathbf{a}^2P_{bb} + \mathbf{a}P_{cc}) + 2(\mathbf{a}P_{ab} + \mathbf{a}^2P_{ac} + P_{bc})]\end{aligned}$$

we find from (*D*):

$$\mathbf{P}_{12} = \tfrac{1}{3}(P_{aa} + \mathbf{a}^2P_{bb} + \mathbf{a}P_{cc}) + \tfrac{2}{3}(\mathbf{a}P_{ab} + \mathbf{a}^2P_{ac} + P_{bc}).$$

Similarly, from

$$\mathbf{E}_{a2} = \tfrac{1}{3}(\mathbf{E}_a + \mathbf{a}^2\mathbf{E}_b + \mathbf{a}\mathbf{E}_c) \text{ follows}$$
$$\mathbf{P}_{22} = \tfrac{1}{3}(P_{aa} + P_{bb} + P_{cc}) - \tfrac{1}{3}(P_{ab} + P_{bc} + P_{ac}), \text{ and from}$$
$$\mathbf{E}_{a0} = \tfrac{1}{3}(\mathbf{E}_a + \mathbf{E}_b + \mathbf{E}_c) \text{ follows}$$
$$\mathbf{P}_{02} = \tfrac{1}{3}(P_{aa} + \mathbf{a}P_{bb} + \mathbf{a}^2P_{cc}) - \tfrac{1}{3}(\mathbf{a}^2P_{ab} + \mathbf{a}P_{ac} + P_{bc}).$$

If only zero sequence charges are present, by definition of the $\mathbf{P}_{m0}$ follows:

$$\mathbf{E}_{a1} = \mathbf{P}_{10}\mathbf{Q}_{a0}, \qquad \mathbf{E}_{a2} = \mathbf{P}_{20}\mathbf{Q}_{a0} \qquad \text{and} \qquad \mathbf{E}_{a0} = \mathbf{P}_{00}\mathbf{Q}_{a0}; \tag{E}$$

or, since $\mathbf{Q}_{a0} = \mathbf{Q}_{b0} = \mathbf{Q}_{c0}$:

$$\mathbf{E}_a = (P_{aa} + P_{ab} + P_{ac})\mathbf{Q}_{a0}$$
$$\mathbf{E}_b = (P_{ab} + P_{bb} + P_{bc})\mathbf{Q}_{a0}$$
$$\mathbf{E}_c = (P_{ac} + P_{bc} + P_{cc})\mathbf{Q}_{a0}.$$

Since

$$\begin{aligned}\mathbf{E}_{a1} &= \tfrac{1}{3}(\mathbf{E}_a + \mathbf{a}\mathbf{E}_b + \mathbf{a}^2\mathbf{E}_c)\\ &= \tfrac{1}{3}\mathbf{Q}_{a0}[P_{aa} + P_{ab} + P_{ac} + \mathbf{a}P_{ab} + \mathbf{a}P_{bb} + \mathbf{a}P_{bc} + \mathbf{a}^2P_{ac} + \mathbf{a}^2P_{bc} + \mathbf{a}^2P_{cc}]\\ &= \tfrac{1}{3}\mathbf{Q}_{a0}[(P_{aa} + \mathbf{a}P_{bb} + \mathbf{a}^2P_{cc}) - (\mathbf{a}^2P_{ab} + \mathbf{a}P_{ac} + P_{bc})],\end{aligned}$$

it follows from (*E*):

$$\mathbf{P}_{10} = \tfrac{1}{3}(P_{aa} + \mathbf{a}P_{bb} + \mathbf{a}^2P_{cc}) - \tfrac{1}{3}(\mathbf{a}^2P_{ab} + \mathbf{a}P_{ac} + P_{bc}).$$

Similarly, we find

$$\mathbf{P}_{20} = \tfrac{1}{3}(P_{aa} + \mathbf{a}^2P_{bb} + \mathbf{a}P_{cc}) - \tfrac{1}{3}(\mathbf{a}P_{ab} + \mathbf{a}^2P_{ac} + P_{bc}) \quad \text{and}$$
$$\mathbf{P}_{00} = \tfrac{1}{3}(P_{aa} + P_{bb} + P_{cc}) + \tfrac{2}{3}(P_{ab} + P_{ac} + P_{bc}).$$

It may be noted that $\mathbf{P}_{10} = \mathbf{P}_{02}$, $\mathbf{P}_{20} = \mathbf{P}_{01}$, $\mathbf{P}_{11} = \mathbf{P}_{22}$ and $\mathbf{P}_{mn} \neq \mathbf{P}_{nm}$. All $\mathbf{P}_{mn}$ $(m \neq n)$ are complex numbers, whereas the P_{ik} are real numbers.

A comparison of the potential coefficients P_{ii} with the P_{ik} $(i \neq k)$ indicates that the P_{ii} are larger than the P_{ik}. A comparison of the sequence potential coefficients $\mathbf{P}_{mm}$ with the complex $\mathbf{P}_{mn}$ $(m \neq n)$ indicates that the P_{mm} are considerably larger than the P_{mn}. Furthermore, since the unbalance is known to be relatively small, the positive sequence charge $\mathbf{Q}_{a1}$ (produced by the impressed positive sequence voltages) will be much larger than either the negative sequence charge $\mathbf{Q}_{a2}$, or the zero sequence charge $\mathbf{Q}_{a0}$ (in the solidly grounded system β), which are both due to the geometric unbalance of the conductor arrangement. Therefore, we may simplify Eqs. *B* by omitting the terms which contain both the relatively small coefficients $\mathbf{P}_{mn}$ $(m \neq n)$ and either $\mathbf{Q}_{a2}$ or $\mathbf{Q}_{a0}$. With balanced line-to-line voltages impressed, the

negative sequence voltage becomes $\mathbf{E}_{a2} = 0$. The simplified Eqs. *B* are then:

$$\begin{aligned} \mathbf{E}_{a1} &= \mathbf{P}_{11}\mathbf{Q}_{a1} = \mathbf{P}_{22}\mathbf{Q}_{a1} \\ \mathbf{E}_{a2} &= \mathbf{P}_{21}\mathbf{Q}_{a1} + \mathbf{P}_{22}\mathbf{Q}_{a2} = 0 \\ \mathbf{E}_{a0} &= \mathbf{P}_{01}\mathbf{Q}_{a1} + \mathbf{P}_{00}\mathbf{Q}_{a0}. \end{aligned} \qquad (F)$$

From Eqs. *F* follow the desired quantities:

$$\mathbf{Q}_{a2} = -\frac{\mathbf{P}_{21}}{\mathbf{P}_{22}}\cdot\mathbf{Q}_{a1}$$

α) $\qquad \mathbf{E}_{a0} = \mathbf{P}_{01}\mathbf{Q}_{a1} \quad$ or $\quad \mathbf{d}_0 = \dfrac{\mathbf{E}_{a0}}{\mathbf{E}_{a1}} = \dfrac{\mathbf{P}_{01}}{\mathbf{P}_{11}} \quad$ with $\mathbf{Q}_{a0} = 0$, or

β) $\qquad \mathbf{Q}_{a0} = -\dfrac{\mathbf{P}_{01}}{\mathbf{P}_{00}}\mathbf{Q}_{a1} \quad$ or $\quad \mathbf{d}'_0 = \dfrac{\mathbf{Q}_{a0}}{\mathbf{Q}_{a1}} = -\dfrac{\mathbf{P}_{01}}{\mathbf{P}_{00}} \quad$ with $\mathbf{E}_{a0} = 0.$ $\qquad (G)$

The same relations exist, of course, between the charging currents $\mathbf{I}'_{a1}$, $\mathbf{I}'_{a2}$ and $\mathbf{I}'_{a0}$ (in the grounded system β).

Similar, but more complex relations can be derived for double circuit lines with or without ground wires.

If $\mathbf{d}'_0$ is defined as $\mathbf{d}'_0 = \dfrac{\mathbf{Q}_{a0}}{\mathbf{Q}_{a1}} = \dfrac{\mathbf{I}'_{a0}}{\mathbf{I}'_{a1}}$, a relation between $\mathbf{d}'_0$ and $\mathbf{d}_0$ (of Appendix 1) can easily be established.

By definition $\mathbf{d}_0 = \dfrac{\mathbf{I}_u}{\mathbf{I}_G} = \dfrac{3\mathbf{I}'_{a0}}{\dfrac{3\mathbf{E}_{a1}}{\mathbf{x}'_0}}$, where $\mathbf{E}_{a1}$ is the phase voltage of the system and $\mathbf{x}'_0$ its zero sequence capacitive shunt reactance.

However $\mathbf{d}'_0 = \dfrac{\mathbf{I}'_{a0}}{\mathbf{I}'_{a1}} = \dfrac{\mathbf{I}'_{a0}}{\dfrac{\mathbf{E}_{a1}}{\mathbf{x}'_1}}$ where $\mathbf{x}'_1$ is the positive sequence capacitive shunt reactance.

From these relations follows

$$\mathbf{d}_0 = \mathbf{d}'_0\frac{\mathbf{x}'_0}{\mathbf{x}'_1} \quad \text{or} \quad \mathbf{d}'_0 = \mathbf{d}_0\frac{\mathbf{x}'_1}{\mathbf{x}'_0}. \qquad (H)$$

APPENDIX 3

MAGNETIC UNBALANCE OF UNTRANSPOSED LINES

Overhead lines serve for the transmission of power from plants to users. Lines are therefore, always terminated at their sending (*S*) and receiving (*R*) ends by balanced equipment, such as transformers and rotating three-phase machines. Basically, magnetic unbalance of lines or line sections means the induction of unbalanced voltages by the currents in the unsymmetrically arranged phase wires. These unbalanced voltages produce negative- and zero-sequence circulating currents. Following Edith Clarke,[5] only the voltage drops $\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$ on the unbalanced line sections need special consideration, because the sequence networks are mutually coupled only on the unbalanced line sections.

Using complete self-($\mathbf{Z}_{iip}$) and mutual-impedances (**4**, **5**) ($\mathbf{Z}_{ikp}$) with ground return ($i, k = a, b, c$; $i \neq k$), these voltages are:

$$\begin{aligned} \mathbf{V}_a &= \mathbf{Z}_{aap}\mathbf{I}_a + \mathbf{Z}_{abp}\mathbf{I}_b + \mathbf{Z}_{acp}\mathbf{I}_c \\ \mathbf{V}_b &= \mathbf{Z}_{abp}\mathbf{I}_a + \mathbf{Z}_{bbp}\mathbf{I}_b + \mathbf{Z}_{bcp}\mathbf{I}_c \\ \mathbf{V}_c &= \mathbf{Z}_{acp}\mathbf{I}_a + \mathbf{Z}_{bcp}\mathbf{I}_b + \mathbf{Z}_{ccp}\mathbf{I}_c. \end{aligned} \qquad (A)$$

In these equations are $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$ the phase currents, and the complete self and mutual impedances with ground return are (**4**, **5**):

$$\begin{aligned} \mathbf{Z}_{iip} &= R_{a(i)} + \tfrac{1}{3}R_e + j(X_{a(i)} + \tfrac{1}{3}X_e) \\ \mathbf{Z}_{ikp} &= \tfrac{1}{3}R_e + j(\tfrac{1}{3}X_e + X_{d(ik)}), \quad i \neq k \end{aligned} \qquad (B)$$

[5] See Ref. 2, pp. 244–247 and 416–417.

with

$$R_e = 0.004,765\,f \text{ ohms/mile } (f = 60 \text{ cycles}), \text{ and}$$

$$X_e = 0.013,970\,f \log 2162 \sqrt{\frac{\rho}{f}} \text{ ohms/mile.}$$

Overhead ground wires ($w, x \cdots$) would add to (A) as many equations as there are ground wires, but since the voltages $\mathbf{V}_w, \mathbf{V}_x \cdots$ are zero, their currents $\mathbf{I}_w, \mathbf{I}_x, \cdots$ may be eliminated, with the result that only three equations in $\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$ remain, just as in Eqs. A of Appendix 2.

Replacing [6] the $\mathbf{Z}_{ikp}$ by self and mutual sequence impedances $\mathbf{Z}_{mn}$ ($m, n = 1, 2, 0$), in the same manner as was done with the potential coefficients $\mathbf{P}_{mn}$ in Appendix 2, leads to a new set of equations:

$$\begin{aligned} \mathbf{V}_{a1} &= \mathbf{Z}_{11}\mathbf{I}_{a1} + \mathbf{Z}_{12}\mathbf{I}_{a2} + \mathbf{Z}_{10}\mathbf{I}_{a0} \\ \mathbf{V}_{a2} &= \mathbf{Z}_{21}\mathbf{I}_{a1} + \mathbf{Z}_{22}\mathbf{I}_{a2} + \mathbf{Z}_{20}\mathbf{I}_{a0} \\ \mathbf{V}_{a0} &= \mathbf{Z}_{01}\mathbf{I}_{a1} + \mathbf{Z}_{02}\mathbf{I}_{a2} + \mathbf{Z}_{00}\mathbf{I}_{a0}. \end{aligned} \qquad (C)$$

Using the same procedure as in Appendix 2, we find expressions for the $\mathbf{Z}_{mn}$, namely:

$$\begin{aligned} \mathbf{Z}_{11} = \mathbf{Z}_{22} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{Z}_{bbp} + \mathbf{Z}_{ccp}) - \tfrac{1}{3}(\mathbf{Z}_{abp} + \mathbf{Z}_{acp} + \mathbf{Z}_{bcp}) \\ \mathbf{Z}_{00} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{Z}_{bbp} + \mathbf{Z}_{ccp}) + \tfrac{2}{3}(\mathbf{Z}_{abp} + \mathbf{Z}_{acp} + \mathbf{Z}_{bcp}) \\ \mathbf{Z}_{12} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{a}^2\mathbf{Z}_{bbp} + \mathbf{a}\mathbf{Z}_{ccp}) + \tfrac{2}{3}(\mathbf{a}\mathbf{Z}_{abp} + \mathbf{a}^2\mathbf{Z}_{acp} + \mathbf{Z}_{bcp}) \\ \mathbf{Z}_{21} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{a}\mathbf{Z}_{bbp} + \mathbf{a}^2\mathbf{Z}_{ccp}) + \tfrac{2}{3}(\mathbf{a}^2\mathbf{Z}_{abp} + \mathbf{a}\mathbf{Z}_{acp} + \mathbf{Z}_{bcp}) \\ \mathbf{Z}_{10} = \mathbf{Z}_{02} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{a}\mathbf{Z}_{bbp} + \mathbf{a}^2\mathbf{Z}_{ccp}) - \tfrac{1}{3}(\mathbf{a}^2\mathbf{Z}_{abp} + \mathbf{a}\mathbf{Z}_{acp} + \mathbf{Z}_{bcp}) \\ \mathbf{Z}_{20} = \mathbf{Z}_{01} &= \tfrac{1}{3}(\mathbf{Z}_{aap} + \mathbf{a}^2\mathbf{Z}_{bbp} + \mathbf{a}\mathbf{Z}_{ccp}) - \tfrac{1}{3}(\mathbf{a}\mathbf{Z}_{abp} + \mathbf{a}^2\mathbf{Z}_{acp} + \mathbf{Z}_{bcp}). \end{aligned} \qquad (D)$$

For lines without ground wires, these sequence impedances can be expressed in simplified forms, since then $\mathbf{Z}_{aap} = \mathbf{Z}_{bbp} = \mathbf{Z}_{ccp}$. Equations B indicate that, in general, Z_{iip} is larger than Z_{ikp} ($i \neq k$). However, the Z_{mm} will be much larger than the Z_{mn} ($m \neq n$) because the operators $\mathbf{a}$ and $\mathbf{a}^2$ in (D) make the Z_{mn} very small.

Considering now the whole transmission system and including the emf's and voltage drops on both sides of the unbalanced line leads to equations:

$$\begin{aligned} \mathbf{E}_{a1} &= (\mathbf{Z}_1 + \mathbf{Z}_{11} + \mathbf{Z}'_1)\mathbf{I}_{a1} + \mathbf{Z}_{12}\mathbf{I}_{a2} + \mathbf{Z}_{10}\mathbf{I}_{a0} + \mathbf{E}'_{a1} \\ 0 &= \mathbf{Z}_{21}\mathbf{I}_{a1} + (\mathbf{Z}_2 + \mathbf{Z}_{22} + \mathbf{Z}'_2)\mathbf{I}_{a2} + \mathbf{Z}_{20}\mathbf{I}_{a0} + 0 \\ 0 &= \mathbf{Z}_{01}\mathbf{I}_{a1} + \mathbf{Z}_{02}\mathbf{I}_{a2} + (\mathbf{Z}_0 + \mathbf{Z}_{00} + \mathbf{Z}'_0)\mathbf{I}_{a0} + 0, \end{aligned} \qquad (E)$$

in which $\mathbf{E}_{a1}$, $\mathbf{Z}_1$, $\mathbf{Z}_2$, and $\mathbf{Z}_0$ are quantities to the left (sending side) of the unbalanced overhead line, and the same but primed quantities indicate contributions on the right (receiving side) of the unbalanced section (see Fig. 4). Since $\mathbf{E}$ and $\mathbf{E}'$ are the generated emf's on the sending

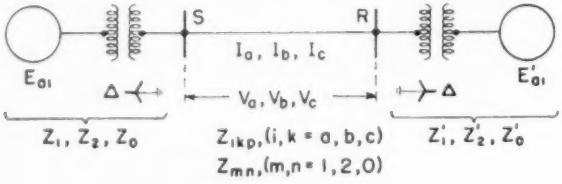

FIG. 4.

(S) and receiving (R) ends $\mathbf{E}_{a1} \equiv E$, $\mathbf{E}_{a2} = \mathbf{E}_{a0} = 0$ and $\mathbf{E}'_{a1} \equiv E'$, $\mathbf{E}_{a2} = \mathbf{E}_{a0} = 0$. Usually $\mathbf{Z}_0$ and $\mathbf{Z}'_0$ will be the zero sequence impedances of only the power transformers since no zero sequence currents can flow into the ungrounded (delta connected) low voltage windings of the transformers.

Considering that $\mathbf{I}_{a2}$ and $\mathbf{I}_{a0}$ will be small relative to $\mathbf{I}_{a1}$, and neglecting all the terms in Eqs. E which are composed of currents $\mathbf{I}_{a2}$ or $\mathbf{I}_{a0}$ multiplied by a very small impedance $\mathbf{Z}_{mn}$ ($m \neq n$)

leads to simplified, approximate equations:

$$\begin{aligned} \mathbf{E}_{a1} &= (\mathbf{Z}_1 + \mathbf{Z}_{11} + \mathbf{Z}'_1)\mathbf{I}_{a1} + \mathbf{E}'_{a1} \\ 0 &= (\mathbf{Z}_2 + \mathbf{Z}_{22} + \mathbf{Z}'_2)\mathbf{I}_{a2} + \mathbf{Z}_{21}\mathbf{I}_{a1} \\ 0 &= (\mathbf{Z}_0 + \mathbf{Z}_{00} + \mathbf{Z}'_0)\mathbf{I}_{a0} + \mathbf{Z}_{01}\mathbf{I}_{a1}, \end{aligned} \qquad (F)$$

from which follow approximate values

$$\mathbf{I}_{a2} = -\frac{\mathbf{Z}_{21}}{\mathbf{Z}_2 + \mathbf{Z}_{22} + \mathbf{Z}'_2}\mathbf{I}_{a1},$$

and

$$\mathbf{I}_{a0} = -\frac{\mathbf{Z}_{01}}{\mathbf{Z}_0 + \mathbf{Z}_{00} + \mathbf{Z}'_0}\mathbf{I}_{a2}. \qquad (G)$$

It may be worthwhile to point out that these Eqs. *G* are quite similar to the Eqs. *G* of Appendix 2. The additional terms $\mathbf{Z}_2$, $\mathbf{Z}'_2$ and $\mathbf{Z}_0$, $\mathbf{Z}'_0$ tend to reduce the resulting current unbalance. The problem here involves a series-connection of the unbalanced line with the symmetrical terminal equipment, whereas the charges of Appendix 2 are shunt loads which are not influenced by other parallel loads.

Similar, but somewhat more complex relations can be derived for double circuit lines with or without ground wires.

High Power Audio Amplifier.—A new variable-frequency audio amplifier (type FG 5/10) capable of delivering either 5 or 10 kw. output power is available from the Westinghouse Electric Corporation. This power amplifier will take a signal of about 10 milliwatts from any conventional 30 to 10,000 cps. source and build it up to 5 or 10 kw. with uniform response (±1.5 db).

The type FG audio amplifier is useful in laboratories where a variable frequency power supply is required or in conjunction with high power pulsing equipment, and also in many industrial processes.

Pocket Magnifier.—A new, inexpensive, 10-power magnifier that can be clipped to the pocket like a fountain pen has been announced by Bausch & Lomb Optical Co., Rochester, N. Y.

The company said the magnifier, which uses two penlight batteries and bulb for illuminating the object, is designed for on-the-job inspection of small parts, castings, small joints and welds, reading micrometer and other fine scales, and similar shop and laboratory uses.

Weighing only two and one-half ounces, including batteries, the magnifier employs a unique lens construction which allows the light from the bulb to go directly through the lens and onto the object, without any direct light reaching the user's eye. Because a 10-power magnifier is most effectively used when held directly to the eye and less than an inch from the object, built-in illumination is highly desirable in order to observe the fine detail for which the magnifier is used.

The lens system is the Bausch & Lomb Coddington, used in a black tenite mount with chrome-plated handle.

Hysteresis Brake.—An improved hysteresis brake that holds constant tension on yarn, wire, etc. by magnetic drag instead of friction has been announced by the Control Department of the General Electric Company.

The brake consists of Alnico magnets so designed and mounted that a magnetic drag (or hysteresis effect) offers a restraining force on a pulley. This force is essentially constant at all speeds and can be adjusted simply by turning the calibrated head mounted above the pulley.

The restraining force of the magnets assures minimum abrasion on the yarn since the material is always in contact with the revolving pulley and is not rubbed or drawn over the surface.

According to G-E engineers, the new brake reverses the direction of rotation a few degrees after stopping, thereby holding stalled tension. The inertia of the brake itself is slight enough that yarn does not become unthreaded during quick acceleration and deceleration.

The construction of the brake permits operational speeds up to 700 yards per minute while tension control ranges from 3.5 to 10.5 grams.

First public showing of the new tension brake was at the Southern Textile Exposition at Greenville, South Carolina, in October.

Applications for the device include tension control for synthetic and natural yarns as well as rubber, glass-fiber, and small diameter wire.

VIBRATIONS OF A CONTINUOUS BEAM UNDER A CONSTANT MOVING FORCE *

BY

EDWARD SAIBEL [1] AND WINSTON F. Z. LEE [1]

INTRODUCTION

The problem of the forced vibrations of beams has been widely studied for many years, but most of the studies have been restricted to single span structures. Regarding the vibrations of continuous beams, the classical approach has been to destroy the continuity at the intermediate supports by cutting the beam at these points. The continuous beam is thus reduced to a series of single span beams, the solution of which can be obtained. The continuity of the beam is restored by equating the deflections, stresses and moments at the supports. The result is a system of linear algebraic equations which must be solved. In the case of free vibrations, the system of equations is homogeneous. The criterion for a non-trivial solution leads to the frequency equation of the continuous beam from which its natural frequencies may be obtained. The system of equations is nonhomogeneous if the continuous beam is subjected to forced vibrations. In principle, the solution may be obtained but it has little practical value because of its intractable form. Consequently some other method of analysis must be found to make the solution of practical value.

One of the authors (1) [2] has developed a solution for the continuous beam which is subjected to a fixed pulsating load. The present paper is an extension of that work to the motion of the beam when acted on by a constant force moving uniformly across the beam as well as to the case where the intermediate supports are either elastic or rigid.

The method used is an extension of the method developed earlier for the solutions of the natural frequencies of continuous beams (2).

It consists essentially of developing the solution in terms of the eigenfunctions and eigenvalues of the beam with the inner constraints removed, referred to as the simple beam. These are presumed known or capable of being found. The restraints are introduced, depending upon the type, in an appropriate manner. It is found most convenient

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

to obtain the differential equations through the use of Lagrange's equations. The complete solution is found and all boundary and initial conditions are satisfied.

THEORY

The usual assumptions made in the classical theory of beam vibrations are assumed in the present paper. Both the damping effects and the mass associated with the exciting loads are neglected.

The differential equation of vibration of a beam under external exciting force $F(x, t)$ is (3)

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y}{\partial x^2}\right] + m(x)\frac{\partial^2 y}{\partial t^2} = F(x, t), \tag{1}$$

where $EI(x)$ is the flexural rigidity of the beam, $m(x)$ is the mass per unit length of the beam, $y(x, t)$ is the deflection.

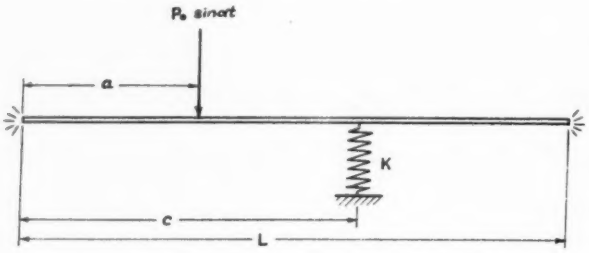

FIG. 1. A continuous two-span beam subjected to a fixed pulsating force.

The deflection $y(x, t)$ may be represented in the form

$$y(x, t) = \sum_{r=1}^{\infty} q_r(t)\phi_r(x) \tag{2}$$

where the q_r's are the generalized coordinates of the vibrating system and ϕ_r's are the eigenfunctions of the corresponding simple beam. These satisfy the following properties:

$$\left.\begin{aligned} \int_0^L m(x)\phi_r(x)\phi_s(x)dx &= 0; \qquad r \neq s \\ \int_0^L m(x)\phi_r^2(x)dx &= 1 \end{aligned}\right\} \tag{3}$$

and form a complete set. Associated with each $\phi_r(x)$, there is an eigenvalue $\lambda_r = p_r^2$ where p_r is the natural circular frequency in the r-th mode of vibration of the simple beam.

FORCED VIBRATION OF A CONTINUOUS TWO-SPAN BEAM UNDER ACTION OF A FIXED, PULSATING AND CONCENTRATED FORCE

The case of the forced vibrations of a continuous two-span beam of uniform section and subjected to a fixed, pulsating and concentrated force ($P_0 \sin \alpha t$) with the forced frequency α acting at a distance a from the left end as shown in Fig. 1 will be considered first. The results will then be extended to a continuous beam of any number of spans of arbitrary lengths. It will be assumed that the intermediate support is elastic having a spring constant K.

To determine the dynamic deflection of the continuous beam under forced vibrations, Lagrange's differential equations of motion are used.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} + \frac{\partial U_c}{\partial q_r} = Q_r \tag{4}$$

where

T = the kinetic energy of the vibrating system,
V = the potential energy of the vibrating system,
U_c = the strain energy stored in the elastic support of spring constant K located at a distance c from the left end of the continuous beam, and
Q_r = the generalized force corresponding to the generalized coordinate q_r.

The kinetic energy of a vibrating system is given by

$$T = \frac{1}{2}\int_0^L m(x)\left(\frac{\partial y}{\partial t}\right)^2 dx. \tag{5}$$

Using (2) and (3), it can be shown that

$$V = \tfrac{1}{2}\sum_{r=1}^{\infty} \dot{q}_r^2. \tag{6}$$

For small deflections of a beam, the potential energy due to bending is

$$V = \frac{1}{2}\int_0^L EI(x)\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx. \tag{7}$$

Substituting Eq. 2 into Eq. 7 and from the orthonormal properties (3) of ϕ_r's, the potential energy of the system becomes

$$V = \tfrac{1}{2}\sum_{r=1}^{\infty} p_r^2 q_r^2. \tag{8}$$

The strain energy U_c stored in the elastic intermediate support is

$$U_c = \tfrac{1}{2}K[y(c, t)]^2 = \tfrac{1}{2}K[\sum_{r=1}^{\infty} q_r\phi_r(c)]^2, \tag{9}$$

where $y(c, t)$ is the deflection of the beam at the intermediate support.

The generalized force Q_r of the fixed pulsating force ($P_0 \sin \alpha t$) can be shown to be

$$Q_r = P_0 \sin \alpha t\, \phi_r(a). \tag{10}$$

Substituting (6), (8), (9) and (10) into (4), the Lagrange's differential equation of motion to be satisfied by each generalized coordinate q_r becomes

$$\ddot{q}_r + p_r^2 q_r + \phi_r(c)[K \sum_{s=1}^{\infty} q_s\phi_s(c)] = P_0\phi_r(a) \sin \alpha t \quad r = 1, 2, 3, \cdots. \tag{11}$$

The complete solution of the linear differential equations (11) will consist of a particular integral and a complementary function. The particular integral yields the solution of the forced vibration while the complementary function gives the solution of the free vibration of the continuous beam.

To find the particular integral of the differential equations (11) let

$$q_r = A_r \sin \alpha t \tag{12}$$

where A_r is a constant to be determined.

Substituting (12) into (11), the following relation is obtained:

$$A_r(p_r^2 - \alpha^2) + \phi_r(c)[K \sum_{s=1}^{\infty} A_s\phi_s(c)] = P_0\phi_r(a). \tag{13}$$

Two different cases may arise: (1) the forced circular frequency α is different from all natural frequencies p_r's of the simple beam, that is, $\alpha \neq p_r$; (2) α coincides with one of the natural frequencies of the simple beam, say $\alpha = p_j$. These will be considered separately.

Case (1): $\alpha \neq p_r$.

Dividing Eq. 13 through by $(p_r^2 - \alpha^2)$ and multiplying the above equation by $\phi_r(c)$ and summing on r, we have

$$\sum_{r=1}^{\infty} A_r\phi_r(c) + \sum_{r=1}^{\infty} \frac{\phi_r^2(c)}{p_r^2 - \alpha^2}[K \sum_{s=1}^{\infty} A_s\phi_s(c)] = P_0 \sum_{r=1}^{\infty} \frac{\phi_r(a)\phi_r(c)}{p_r^2 - \alpha^2}.$$

Since $y(c, t) = \sum_{r=1}^{\infty} q_r\phi_r(c) = \sin \alpha t \sum_{r=1}^{\infty} A_r\phi_r(c)$, $\sum_{r=1}^{\infty} A_r\phi_r(c)$ is the maximum deflection at the elastic intermediate support and $K \sum_{r=1}^{\infty} A_r\phi_r(c)$ is the corresponding maximum support reaction.

Let

$$\left.\begin{aligned} Y(c) &= \sum_{r=1}^{\infty} A_r\phi_r(c) \\ \text{and} \qquad & \\ R(c) &= K \sum_{r=1}^{\infty} A_r\phi_r(c) \end{aligned}\right\} \tag{14}$$

then the above equations become

$$Y(c) + R(c) \sum_{r=1}^{\infty} \frac{\phi_r^2(c)}{p_r^2 - \alpha^2} = P_0 \sum_{r=1}^{\infty} \frac{\phi_r(a)\phi_r(c)}{p_r^2 - \alpha^2}. \tag{15}$$

In case the intermediate support is rigid, the spring constant K of the intermediate support becomes infinite as the deflection $Y(c)$ at the intermediate support becomes zero. The product however remains finite and is $R(c)$ which by Eq. 15 becomes

$$R(c) = P_0 \left\{ \frac{\sum_{r=1}^{\infty} \frac{\phi_r(a)\phi_r(c)}{p_r^2 - \alpha^2}}{\sum_{r=1}^{\infty} \frac{\phi_r^2(c)}{p_r^2 - \alpha^2}} \right\}. \tag{16}$$

Let y_p be the deflection obtained from the particular integral, then Eq. 2 becomes

$$\begin{aligned} y_p(x, t) &= \sin \alpha t \sum_{r=1}^{\infty} A_r\phi_r(x) \\ &= \sin \alpha t \sum_{r=1}^{\infty} \frac{P_0\phi_r(a) - R(c)\phi_r(c)}{p_r^2 - \alpha^2} \phi_r(x) \qquad \text{for} \quad \alpha \neq p_r \end{aligned} \tag{17}$$

where $R(c)$ is given by (16).

Case (2): Let $\alpha = p_j$.

From Eq. 13, for $r = j$, $(p_r^2 - \alpha^2) = (p_j^2 - \alpha^2) = 0$, then

$$R(c) = P_0 \left[\frac{\phi_j(a)}{\phi_j(c)} \right], \qquad \text{if} \qquad \phi_j(c) \neq 0. \tag{18}$$

A_j is not determined here. For $r \neq j$, expressions are the same as in case (1). A_j is determined by the constraint condition at the rigid intermediate support; namely,

$$y_p(c, t) = 0 = \sin \alpha t \left[A_j \phi_j(c) + \sum_{r=1}^{\infty}{}' A_r \phi_r(c)\right]$$

for all time t, where $\sum_{r=1}^{\infty}{}'$ means sum over r with $r = j$ omitted.

Thus

$$A_j = -\sum_{r=1}^{\infty}{}' P_0 \left[\frac{\phi_r(a) - \dfrac{\phi_j(a)}{\phi_j(c)}\phi_r(c)}{p_r^2 - \alpha^2}\right]\frac{\phi_r(c)}{\phi_j(c)} \qquad \text{where} \qquad \phi_j(c) \neq 0$$

and

$$y_p(x, t) = P_0 \sin \alpha t \left\{\sum_{r=1}^{\infty}{}' \left[\frac{\phi_r(a) - \dfrac{\phi_j(a)}{\phi_j(c)}\phi_r(c)}{p_r^2 - \alpha^2}\right]\left[\phi_r(x) - \frac{\phi_r(c)}{\phi_j(c)}\phi_j(x)\right]\right\} \tag{19}$$

for $\alpha = p_j$ and $\phi_j(c) \neq 0$.

If $\alpha = p_j$ and $\phi_j(c) = 0$, the value of A_j is indeterminate. It will be shown later that this condition leads to the case where the free vibration frequency of the simple beam p_j coincides with one of the free vibration frequencies of the continuous beam.

FORCED VIBRATION OF A CONTINUOUS TWO-SPAN BEAM SUBJECTED TO A CONSTANT MOVING FORCE

Instead of a concentrated stationary and pulsating force as treated above, consider the problem of the vibrations of the same continuous two-span beam but subjected to a transverse concentrated force of constant magnitude P moving with uniform velocity v as shown in Fig. 2. Let the constant moving force start at the left end at $t = 0$ and proceed from left to right. Its position at time t will be given by $x = vt$.

The constant moving force P may be expressed by the following Fourier sine series (4):

$$\frac{2P}{L}\left[\sin\frac{\pi x}{L}\sin\omega_1 t + \sin\frac{2\pi x}{L}\sin\omega_2 t + \cdots + \sin\frac{k\pi x}{L}\sin\omega_k t + \cdots\right] \tag{20}$$

where

$$\omega_k = \frac{k\pi v}{L} \qquad k = 1, 2, 3, \cdots.$$

Equation 20 shows that a constant moving force P is in effect equivalent to an infinite number of stationary but pulsating load distributions of sinusoidal character.

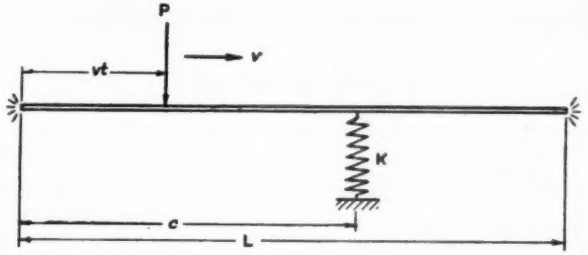

FIG. 2. A continuous two-span beam subjected to a constant moving force.

Since the differential equation representing the vibration of the beam is linear, the principle of superposition holds. The deflection of the beam subjected to the constant moving force can be obtained by the superposition of the deflections produced by the stationary pulsating load distributions in Eq. 20.

Consider the continuous two-span beam subjected to a stationary pulsating distributed load $\left[\frac{2P}{L}\sin\frac{k\pi x}{L}\sin\omega_k t\right]$ with circular frequency $\omega_k = \frac{k\pi v}{L}$. The forced vibration of the continuous two-span beam subjected to a fixed pulsating concentrated force ($P_0 \sin \alpha t$) has been determined and is expressed by (17) or (19). To find the forced vibration due to the stationary pulsating distributed force $\left[\frac{2P}{L}\sin\frac{k\pi x}{L}\sin\omega_k t\right]$ replace ($P_0 \sin \alpha t$) by $\left(\frac{2P}{L}\sin\frac{k\pi a}{L}da\right)\sin\omega_k t$ and α by ω_k in Eq. 17 or

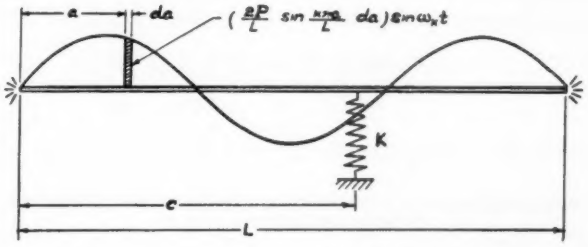

FIG. 3. A continuous two-span beam subjected to a fixed pulsating load of sinusoidal distribution.

19 as shown in Fig. 3. The resulting equation is then integrated with respect to a from $a = 0$ to $a = L$.

Consider the case where ω_k is different from any of the natural frequencies, p_r, of the simple beam. Let $(y_p)_k$ be the deflection of the forced vibration of the continuous beam due to the load

$$\left[\frac{2P}{L}\sin\frac{k\pi x}{L}\sin\omega_k t\right].$$

Then

$$(y_p)_k(x, t) = \frac{2P}{L}\sin\omega_k t$$

$$\times\left\{\sum_{r=1}^{\infty}\left(\frac{\phi_r(x)}{p_r^2 - \omega_k^2}\right)\int_0^L\left[\phi_r(a)\sin\frac{k\pi a}{L} - \phi_r(c)\bar{R}_k(c)\sin\frac{k\pi a}{L}\right]da\right\} \quad (21)$$

where $\overline{R_k}(c)$ is given by

$$\overline{R_k}(c) = \left\{\frac{\sum_{s=1}^{\infty}\frac{\phi_s(c)\phi_s(a)}{p_s^2 - \omega_k^2}}{\sum_{s=1}^{\infty}\frac{\phi_s^2(c)}{p_s^2 - \omega_k^2}}\right\}.$$

Define

$$S_{rk} = \int_0^L\left[\phi_r(a)\sin\frac{k\pi a}{L} - \phi_r(c)\overline{R_k}(c)\sin\frac{k\pi a}{L}\right]da. \quad (22)$$

Equation 21 may then be written

$$(y_p)_k(x, t) = \frac{2P}{L}\sin\omega_k t\left\{\sum_{r=1}^{\infty}\frac{S_{rk}\phi_r(x)}{p_r^2 - \omega_k^2}\right\} \quad (23)$$

S_{rk} can be readily evaluated when the eigenfunctions of the simple beams are known.

Let $y_p(x, t)$ be the forced vibration of the continuous two-span beam subject to a constant moving force. By superposition

$$y_p(x, t) = \frac{2P}{L}\sum_{k=1}^{\infty}\sin\omega_k t\sum_{r=1}^{\infty}\left[\frac{S_{rk}\phi_r(x)}{p_r^2 - \omega_k^2}\right] \quad (24)$$

where none of the ω_k's equals any of the p_r's.

FREE VIBRATIONS OF A CONTINUOUS TWO-SPAN BEAM SUBJECTED TO A CONSTANT MOVING FORCE

The free vibrations of the continuous beam are determined independently of the external load and are given by the complementary

function of the differential Eq. 11, namely,

$$\ddot{q}_r + p_r^2 q_r + \phi_r(c)[K \sum_{s=1}^{\infty} q_s \phi_s(c)] = 0. \tag{25}$$

Let the continuous beam vibrate with one of its natural frequencies Λ_n and let

$$q_r = B_{rn} \sin (\Lambda_n t + e_n) \tag{26}$$

where B_{rn} is a constant to be determined and e_n is the phase angle, also to be determined.

Substituting (26) into (25) and considering only the case where Λ_n is different from any of the p_r's,

$$(p_r^2 - \Lambda_n^2) B_{rn} + \phi_r(c) R_n(c) = 0 \tag{27}$$

where $R_n(c)$, defined as $K \sum_{s=1}^{\infty} B_{sn} \phi_s(c)$, is the maximum support reaction of the intermediate support due to the free vibration of the continuous beam with the particular natural frequency Λ_n.

The deflection $(y_f)_n(x, t)$ due to the free vibration of natural frequency Λ_n is thus

$$\begin{aligned}(y_f)_n(x, t) &= \sum_{r=1}^{\infty} q_r \phi_r(x) \\ &= - R_n(c) \sin (\Lambda_n t + e_n) \sum_{r=1}^{\infty} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_n^2}.\end{aligned} \tag{28}$$

The total deflection $y_f(x, t)$ due to the free vibration of the continuous beam is obtained by superposition as

$$y_f(x, t) = - \sum_{n=1}^{\infty} R_n(c) \sin (\Lambda_n t + e_n) \sum_{r=1}^{\infty} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_n^2}. \tag{29}$$

Let $y(x, t)$ be the total deflection of the vibrating beam under the constant moving force, then

$$y(x, t) = y_p(x, t) + y_f(x, t)$$

or

$$\begin{aligned}y(x, t) = \frac{2P}{L} \left\{ \sum_{k=1}^{\infty} \sin \omega_k t \sum_{r=1}^{\infty} \frac{S_{rk}\phi_r(x)}{p_r^2 - \omega_k^2} \right\} \\ - \sum_{n=1}^{\infty} R_n(c) \sin (\Lambda_n t + e_n) \sum_{r=1}^{\infty} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_n^2}\end{aligned} \tag{30}$$

where the S_{rk}'s are given by (22).

The $R_n(c)$'s and e_n's are to be determined from the initial conditions of the vibrating system.

Without essential loss in generality it is assumed that both the deflection and velocity of the beam are zero at $t = 0$.

(i) $y(x, 0) = 0$ for $0 \leq x \leq L$ requires $e_n = 0$

(ii) $\dfrac{\partial y}{\partial t} = 0$ at $t = 0$ for $0 \leq x \leq L$

leads to

$$\sum_{n=1}^{\infty} \frac{\Lambda_n}{p_r^2 - \Lambda_n^2} R_n(c) = \frac{2P}{L} \cdot \frac{1}{\phi_r(c)} \sum_{k=1}^{\infty} \frac{\omega_k}{p_r^2 - \omega_k^2} S_{rk} \tag{31}$$

where $r = 1, 2, 3, \cdots$.

Equations 31 form a nonhomogeneous system of infinitely many equations in infinitely many unknowns $R_n(c)$'s from which the $R_n(c)$'s may be solved to a degree of approximation as closely as desired.

Thus the total deflection of the continuous two-span beam subjected to a moving constant force P travelling with uniform velocity v is

$$y(x, t) = \left[\frac{2P}{L} \sum_{k=1}^{\infty} \sin \omega_k t \sum_{r=1}^{\infty} \frac{S_{rk}\phi_r(x)}{p_r^2 - \omega_k^2} \right] - \left[\sum_{n=1}^{\infty} R_n(c) \sin \Lambda_n t \sum_{r=1}^{\infty} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_n^2} \right] \tag{32}$$

where S_{rk} and $R_n(c)$ are given by (22) and (31), respectively.

SPECIAL CASES

Referring to solution (32) for the dynamic deflection of a continuous beam under constant moving load, the following two cases require special treatment.

(1) One of the natural frequencies of the continuous beam, say Λ_m, equals any one of the natural frequencies of the simple beam, p_j, where none of them coincides with any one of the forced frequencies ω_k. Let this case be represented by $\Lambda_m = p_j \neq \omega_k$.

(2) One of the forced frequencies, say ω_m, equals any one of the natural frequencies of the simple beam, p_j, where none of them coincides with any one of the natural frequencies Λ_n of the continuous beam. Let this case be represented by $\omega_m = p_j \neq \Lambda_n$.

Case (1): $\Lambda_m = p_j \neq \omega_k$.

For the free vibration of the continuous beam with natural frequency Λ_m, Eq. 27 gives

$$(p_r^2 - \Lambda_m^2)B_{rm} + \phi_r(c)R_m(c) = 0. \tag{33}$$

For $r = j$, $p_j^2 = \Lambda_m^2$ by assumption, (33) becomes

$$R_m(c)\phi_j(c) = 0. \tag{34}$$

B_{jm} is not determined by (33).

From Eq. 34 either $R_m(c) = 0$ or $\phi_j(c) = 0$, or both. If $R_m(c) = 0$ and $\phi_j(c) \neq 0$, then from (33) it follows that all coefficients B_{rm} except B_{jm} are equal to zero. Upon substitution in (28) this leaves

$$(y_f)_m(x, t) = B_{jm}\phi_j(x) \sin (\Lambda_m t + e_m).$$

The restraint condition at intermediate support $x = c$ requires

$$(y_f)_m(c, t) = B_{jm}\phi_j(c) \sin (\Lambda_m t + e_m) = 0$$

or $B_{jm} = 0$.

This leads to the trivial case $(y_f)_m(x, t) = 0$.

If $\phi_j(c) = 0$ and $R_m(c) \neq 0$, Eq. 33 yields

$$B_{rm} = -\frac{R_m(c)\phi_r(c)}{p_r^2 - \Lambda_m^2} \qquad \text{where} \qquad r \neq j.$$

B_{jm} is not determined.

The deflection due to the free vibration in the m-th mode where $\Lambda_m = p_j$ becomes

$$(y_f)_m(x, t) = \left\{B_{jm}\phi_j(x) - R_m(c) \sum_{r=1}^{\infty}{}^{j} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_m^2}\right\} \sin \Lambda_m t. \tag{35}$$

The values of B_{jm} and $R_m(c)$ are determined by using the initial conditions for the total deflection of the continuous beam and are found to be

$$B_{jm} = -\frac{2P}{L} \sum_{s=1}^{\infty} \frac{\frac{\omega_s}{\Lambda_m} S_{js}}{p_j^2 - \omega_s^2} \tag{36}$$

and

$$\sum_{n=1}^{\infty} \frac{\Lambda_n}{p_r^2 - \Lambda_n^2} R_n(c) = \frac{2P}{L} \sum_{s=1}^{\infty} \frac{\frac{\omega_s}{\phi_r(c)} S_{rs}}{p_r^2 - \omega_s^2} \tag{37}$$

where $r = 1, 2, 3, \cdots$ with $r = j$ omitted. The case where both $\phi_j(c) = 0$ and $R_m(c) = 0$ is just a particular case of the previous one, in which coefficients $B_{rm} = 0$ except B_{jm}. Thus $\phi_j(c) = 0$ is the condition for the natural frequency p_j of the simple beam to be equal to one of the natural frequencies of the continuous beam. The total deflection of the continuous beam under constant moving load when

$\Lambda_m = p_j \neq \omega_k$ is

$$y(x, t) = \frac{2P}{L}\left\{\left[\sum_{k=1}^{\infty} \sin \omega_k t \sum_{r=1}^{\infty} \frac{S_{rk}\phi_r(x)}{p_r^2 - \omega_k^2}\right] - \left[\sin \Lambda_m t \sum_{k=1}^{\infty} \frac{\frac{\omega_k}{\Lambda_m} S_{jk}\phi_j(x)}{p_j^2 - \omega_k^2}\right]\right\}$$

$$- \left\{\sum_{n=1}^{\infty} R_n(c) \sin \Lambda_n t \sum_{r=1}^{k} \frac{\phi_r(c)\phi_r(x)}{p_r^2 - \Lambda_n^2}\right\} \quad (38)$$

where

$$S_{rk} = \int_0^L \left[\phi_r(a) \sin \frac{k\pi a}{L} - \phi_r(c)\overline{R_k}(c) \sin \frac{k\pi a}{L}\right] da$$

and $R_n(c)$'s are given by Eqs. 36.

Case (2): $\omega_m = p_j \neq \Lambda_n$.

When $\omega_m = p_j \neq \Lambda_n$ the deflection of the continuous beam as given by solution (32) becomes indeterminate.

Equation 19 gives the force deflection of a continuous two-span beam subjected to a fixed pulsating force ($P_0 \sin \omega_m t$) at a distance a from the left end when $\omega_m = p_j \neq \Lambda_n$ as

$$y_p(x, t) = P_0 \sin \omega_m t \left\{\sum_{r=1}^{j} \left[\frac{\phi_r(a) - \frac{\phi_j(a)}{\phi_j(c)}\phi_r(c)}{p_r^2 - \omega_m^2}\right]\left[\phi_r(x) - \frac{\phi_r(c)}{\phi_j(c)}\phi_j(x)\right]\right\}.$$

The deflection due to the forced vibration of the continuous beam under the particular pulsating distributed load $\left[\frac{2P}{L} \sin \frac{m\pi x}{L} \sin \omega_m t\right]$ with forced frequency $\omega_m = \frac{m\pi v}{L}$ is

$$(y_p)_m(x, t) = \frac{2P}{L} \sin \omega_m t \left\{\sum_{r=1}^{j}\left[\phi_r(x) - \frac{\phi_r(c)}{\phi_j(c)}\phi_j(x)\right]\right.$$

$$\left. \times \frac{\left[\int_0^L \left(\phi_r(a) \sin \frac{m\pi a}{L} - \frac{\phi_r(c)}{\phi_j(c)}\phi_j(a) \sin \frac{m\pi a}{L}\right) da\right]}{(p_r^2 - \omega_m^2)}\right\}.$$

Let

$$S'_{rm} = \int_0^L \left[\phi_r(a) \sin \frac{m\pi a}{L} - \frac{\phi_r(c)}{\phi_j(c)}\phi_j(a) \sin \frac{m\pi a}{L}\right] da \quad (39)$$

then the above solution may be written

$$(y_p)_m(x, t) = \frac{2P}{L} \sin \Lambda_m t \left\{ \sum_{r=1}^{\infty}{}^{j} \frac{S'_{rm}}{p_r^2 - \omega_m^2} \left[\phi_r(x) - \frac{\phi_r(c)}{\phi_j(c)} \phi_j(x) \right] \right\}. \quad (40)$$

The total deflection of the continuous two-span beam under the constant moving load when $\omega_m = p_j \neq \Lambda_n$ is

$$\begin{aligned} y(x, t) = \frac{2P}{L} \Bigg\{ & \sin \omega_m t \sum_{r=1}^{\infty}{}^{j} \frac{S'_{rm}}{p_r^2 - \omega_m^2} \left[\phi_r(x) - \frac{\phi_r(c)}{\phi_j(c)} \phi_j(x) \right] \\ & + \sum_{s=1}^{\infty}{}^{m} \sin \omega_s t \sum_{r=1}^{\infty} \frac{S_{rs} \phi_r(x)}{p_r^2 - \omega_s^2} \Bigg\} \\ & - \left\{ \sum_{n=1}^{\infty} R_n(c) \sin (\Lambda_n t) \sum_{r=1}^{\infty} \frac{\phi_r(c) \phi_r(x)}{p_r^2 - \Lambda_n^2} \right\} \end{aligned} \quad (41)$$

where the $R_n(c)$'s are determined from the initial conditions, leading to

$$\left.\begin{aligned} \sum_{n=1}^{\infty} \frac{\Lambda_n}{p_j^2 - \Lambda_n^2} R_n(c) &= \frac{2P}{L\phi_j(c)} \left[\sum_{s=1}^{\infty}{}^{m} \frac{\omega_s S_{js}}{p_j^2 - \omega_s^2} - \sum_{s=1}^{\infty}{}^{j} \frac{\omega_m \dfrac{\phi_s(c)}{\phi_j(c)} S'_{sm}}{p_s^2 - \omega_m^2} \right] \\ \sum_{n=1}^{\infty} \frac{\Lambda_n}{p_r^2 - \Lambda_n^2} R_n(c) &= \frac{2P}{L\phi_r(c)} \left[\sum_{s=1}^{\infty}{}^{m} \frac{\omega_s S_{rs}}{p_r^2 - \omega_s^2} + \frac{\omega_m S'_{rm}}{p_r^2 - \omega_m^2} \right] \end{aligned}\right\} \quad (42)$$

where $r = 1, 2, \cdots$, with $r = j$ omitted, and S'_{rm} and S_{rm} are given by (39) and (22) respectively.

STRESS CALCULATION

Let $\sigma(x, t)$ be the maximum dynamic flexural stress at a distance x from the left end of the continuous beam and at any time t.

For small deflections

$$\sigma(x, t) = -\frac{EI}{Z} \frac{\partial^2 y}{\partial x^2} \quad (43)$$

where Z is the section modulus of the beam. Once the deflection $y(x, t)$ has been found, the stress may be calculated in straight forward manner, the solutions obtained being differentiable two times.

CONTINUOUS BEAM OF ANY NUMBER OF SPANS OF ARBITRARY LENGTHS

The above results for a continuous two-span beam can be readily extended to a continuous beam of $(n + 1)$ span of arbitrary lengths subjected to a constant moving force. Let subscripts $1, 2, \cdots, n$ be attributed to quantities of the 1st, 2nd, $\cdots$, n-th intermediate rigid supports at distances $c_1, c_2, \cdots, c_n$, respectively, from the left end of

the beam. It can be shown that the total deflection is

$$y(x, t) = \left\{ \frac{2P}{L} \sum_{k=1}^{\infty} \sin \omega_k t \left[\sum_{r=1}^{\infty} \frac{\phi_r(x)}{p_r^2 - \omega_k^2} \times \int_0^L \left(\phi_r(a) \sin \frac{k\pi a}{L} - \sum_{i=1}^{n} \phi_r(c_i) \overline{R_k}(c_i) \sin \frac{k\pi a}{L} \right) da \right] \right\} - \left\{ \sum_{m=1}^{\infty} \sin \Lambda_m t \sum_{r=1}^{\infty} \frac{\sum_{i=1}^{n} \phi_r(c_i) R_m(c_i)}{p_r^2 - \Lambda_m^2} \phi_r(x) \right\} \quad (44)$$

where

$$\overline{R_k}(c_i) = \frac{\Delta_1}{\Delta_2}, \quad (45)$$

and Δ_2 is the determinant $\left| \sum_{r=1}^{\infty} \frac{\phi_r(c_s)\phi_r(c_t)}{p_r^2 - \omega_k^2} \right|$, $s, t = 1, 2, \cdots n$.

Δ_1 is the same as Δ_2 except that the i-th column is replaced by

$$\sum_{r=1}^{\infty} \frac{\phi_r(c_s)\phi_r(a)}{p_r^2 - \omega_k^2} \qquad (s = 1, 2, \cdots n)$$

and $R_m(c_i)$'s are given by the following equations:

$$\sum_{m=1}^{\infty} \frac{\Lambda_m}{p_r^2 - \Lambda_m^2} \sum_{i=1}^{n} \phi_r(c_i) R_m(c_i) = \frac{2P}{L} \sum_{k=1}^{\infty} \frac{\omega_k}{p_r^2 - \omega_k^2} \times \left\{ \int_0^L \left[\phi_r(a) \sin \frac{k\pi a}{L} - \sum_{i=1}^{n} \phi_r(c_i) \overline{R_k}(c_i) \sin \frac{k\pi a}{L} \right] da \right\} \quad (46)$$

where $r = 1, 2, 3, \cdots$.

The dynamic flexural stress $\sigma(x, t)$ follows readily

$$\sigma(x, t) = -\frac{EI}{Z} \left\{ \left[\frac{2P}{L} \sum_{k=1}^{\infty} \sin \omega_k t \sum_{r=1}^{\infty} \frac{\frac{d^2}{dx^2} \phi_r(x)}{p_r^2 - \omega_k^2} \times \int_0^L \left(\phi_r(a) \sin \frac{k\pi a}{L} - \sum_{i=1}^{n} \phi_r(c_i) \overline{R_k}(c_i) \sin \frac{k\pi a}{L} \right) da \right] - \left[\sum_{m=1}^{\infty} \sin \Lambda_m t \sum_{r=1}^{\infty} \frac{\sum_{i=1}^{n} \phi_r(c_i) R_m(c_i)}{p_r^2 - \Lambda_m^2} \frac{d^2}{dx^2} \phi_r(x) \right] \right\}. \quad (47)$$

For special cases where any one of the natural frequencies of the simple beam coincides with either a forced frequency of the constant moving force or a natural frequency of the continuous beam of n-spans, results can be obtained similarly as above from the extension of those of continuous two-span beam.

NUMERICAL EXAMPLE

For numerical calculations, the ratios of span lengths do not affect appreciably the amount of labor involved. However, the case of a continuous beam consisting of two equal spans simply supported at the ends was selected because experimental results were available (5). A theoretical solution to the dynamic flexural stress of this particular case of two equal spans has also been given by Ayre, Ford and Jacobsen (5). Their method is apparently not amenable to generalization, such as varying span ratios or introducing more intermediate supports. The present method is not subjected to such limitations.

As a numerical example, consider a two-span beam of equal span lengths, that is, $c = L/2$ simply supported at two ends. Take $\frac{\omega_1}{p_1} = 1.322$ (this corresponding to a value of 0.661 in the notation of reference (5)).

The eigenfunctions for the simple beam with both ends simply supported are

$$\phi_r(x) = \sqrt{\frac{2}{Lm}} \sin \frac{r\pi x}{L}$$

and the corresponding eigenvalues are

$$\lambda_r = p_r^2 = r^4 p_1^2 = r^4 \frac{\pi^4 EI}{L^4 m}.$$

For the continuous beam under consideration, its odd numbered natural frequencies $\Lambda_{(2n-1)}$ $(n = 1, 2, \cdots)$ coincide with the corresponding even numbered natural frequencies of the simple beam p_{2n} $(n = 1, 2, \cdots)$, *viz.*

$$\Lambda_{(2n-1)} = p_{2n} \qquad \text{where} \qquad n = 1, 2, 3, \cdots.$$

This corresponds to special case (1) where $\Lambda_m = p_j \neq \omega_k$. Substituting above quantities into Eqs. 38 and 43 it can be shown that

$$\sigma(x, t) = \frac{2PL}{\pi^2 Z} \left\{ \sum_{k=1}^{\infty} \frac{\sin \omega_k t}{k^2 \left(1 - \frac{\omega_k^2}{p_k^2}\right)} \left\{ \sin \frac{k\pi x}{L} - \frac{1}{k^2} \sin \frac{k\pi c}{L} \frac{\sum_{s=1}^{\infty} \frac{\sin \frac{s\pi c}{L} \sin \frac{s\pi x}{L}}{s^2 \left(1 - \frac{\omega_k^2}{p_s^2}\right)}}{\sum_{s=1}^{\infty} \frac{\sin^2 \frac{s\pi c}{L}}{s^4 \left(1 - \frac{\omega_k^2}{p_s^2}\right)}} \right\} \right.$$

(*Equation continued on next page.*)

$$- \sum_{m=1}^{\infty} \frac{R_m(c)}{P} \sin \Lambda_m t \sum_{n=1}^{\infty} \frac{\left[\sin \frac{(2n-1)\pi c}{L}\right]\left[\sin \frac{(2n-1)\pi x}{L}\right]}{(2n-1)^2\left[1 - \frac{\Lambda_m^2}{p_{(2n-1)}^2}\right]}$$

$$\left. - \sum_{n=1}^{\infty} \frac{\omega_{2n}}{\Lambda_{(2n-1)}} \frac{\sin \frac{2n\pi x}{L} \sin \Lambda_{(2n-1)} t}{(2n)^2\left[1 - \left(\frac{\omega_{2n}}{p_{2n}}\right)^2\right]} \right\} \tag{48}$$

where $R_m(c)$'s are given by

$$\sum_{m=1}^{\infty} \frac{\Lambda_m}{\left[1 - \left(\frac{\Lambda_m}{p_{(2n-1)}}\right)^2\right]} \frac{R_m(c)}{P}$$

$$= \left\{ \frac{\omega_{(2n-1)}}{\sin \frac{(2n-1)\pi c}{L}} \cdot \frac{1}{\left(1 - \frac{\omega_{(2n-1)}^2}{p_{(2n-1)}^2}\right)} - \sum_{k=1}^{\infty} \frac{\frac{\omega_k \sin \frac{k\pi c}{L}}{k^4\left(1 - \frac{\omega_k^2}{p_k^2}\right)\left(1 - \frac{\omega_k^2}{p_{(2n-1)}^2}\right)}}{\sum_{s=1}^{\infty} \frac{\sin^2 \frac{s\pi c}{L}}{s^4\left(1 - \frac{\omega_k^2}{p_s^2}\right)}} \right\} \tag{49}$$

where $n = 1, 2, 3, \cdots$.

Let σ_s be the flexural stress at the lower surface of the mid-point of the left span of the continuous beam with two equal spans subjected to a static load P acting at the same point where σ_s is considered. It is found that

$$\sigma_s = 1.0024 \frac{PL}{\pi^2 Z}. \tag{50}$$

The stress ratio $\frac{\sigma}{\sigma_s}$ at points $x = L/4$ and $x = 3L/4$, where the maximum stress is most likely to occur, has been calculated for $vt = 0$, $L/4$, $L/2$, $3L/4$, and L. These results are readily obtained from Eqs. 48 and 49 and are summarized in the following table:

Stress Ratios at $x = L/4$ and $x = 3L/4$ for a Continuous Beam of Two Equal Spans Subjected to a Moving Constant Force.

vt	0	$L/4$	$L/2$	$3L/4$	L
$\left(\frac{\sigma}{\sigma_s}\right)_{x=L/4}$	0	0.9900	0.2613	−0.8208	−0.1381
$\left(\frac{\sigma}{\sigma_s}\right)_{x=3L/4}$	0	−0.0860	−0.9005	1.9910	−0.2530

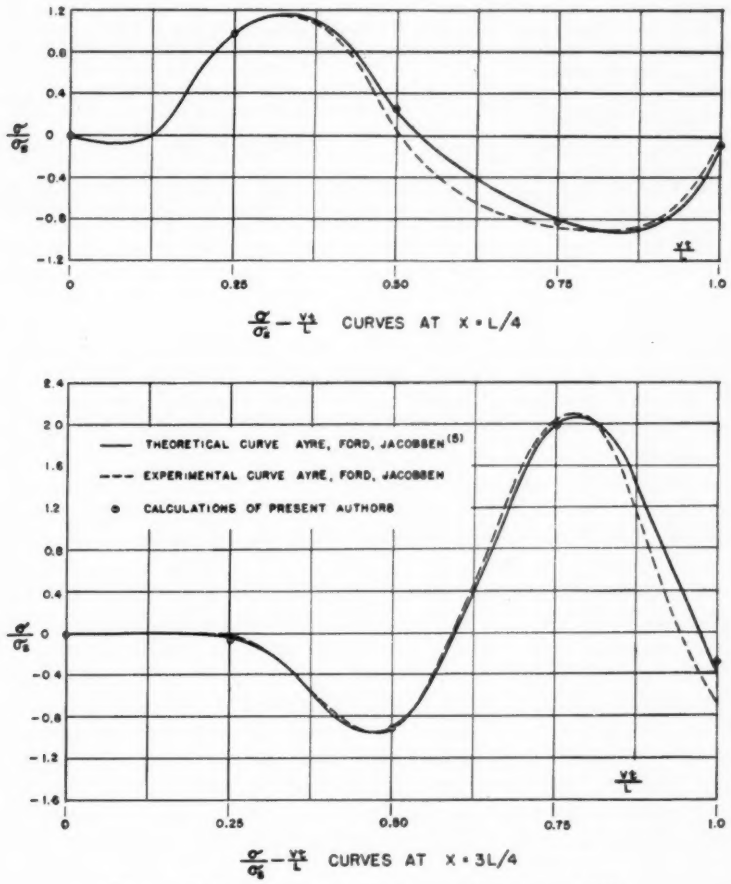

FIG. 4. Ratio of dynamic to static stress for $\omega_1/p_1 = 1.332$.

From Fig. 4, it is seen that the results are in very good agreement with both the experimental and theoretical values given in ref. (5). The numerical computations were simple and straight forward. For

each value of vt, the first five non-zero terms for forced vibrations and the first three terms for free vibrations are taken into account in the calculations of the values shown in the above table.

The above results indicate that the maximum dynamic stress is about twice as great as the static stress even when the vibration is not in resonance. A greater magnification of stress is expected when resonance is approached.

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A FERROELECTRIC AMPLIFIER

BY

HARRY URKOWITZ[1]

ABSTRACT

"Ferroelectric" refers to a class of nonlinear dielectrics characterized by a charge *vs.* voltage relationship exhibiting hysteresis and dielectric saturation. A capacitor made from barium-strontium titanate ceramic, a ferroelectric material, was used as the capacitive element in a single tuned circuit to which a high frequency current was applied. A low frequency signal to be amplified was applied to the capacitor, causing its capacitance to vary. The resulting amplitude-modulated high frequency voltage was applied to a detector circuit to recover the signal frequency. An analysis of this circuit shows what must be done to achieve maximum amplification. A circuit such as described was constructed and power gains of about 60 were obtained.

When the signal was removed and the output of the amplifier fed back to the input, sustained oscillations of low frequency were obtained, with only the high frequency current source applied. The frequency of oscillation could be varied continuously over a wide range.

The word "ferroelectric" refers to a certain class of dielectric materials. These materials are nonlinear, having dielectric coefficients which vary with applied voltage. If a capacitor is made of such a material and a plot made of charge *versus* applied voltage, the curve will show a striking resemblance to the *B–H* curves of ferromagnetic materials, exhibiting dielectric saturation and hysteresis, hence the term "ferroelectric." Until recently the only materials known to have this property were certain piezoelectric crystals among which was Rochelle salt.

The development of high dielectric coefficient ceramics consisting of barium titanate and mixtures of barium and strontium titanates provided another class of ferroelectric materials. The Laboratory for Insulation Research of the Massachusetts Institute of Technology has made an active study of these materials. While working in this laboratory, Shepard Roberts[2] undertook an exhaustive investigation of the properties of barium titanate and barium-strontium titanate. Besides the investigation of the properties of these materials, Roberts gives several possible applications, one of which is a dielectric amplifier. He provides a schematic but does not state whether such an amplifier was ever constructed.

The purpose of the research reported in this paper was to see whether

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[2] S. ROBERTS, "Barium Titanate and Barium-Strontium Titanate as Nonlinear Dielectrics," Tech. Report No. 1, Laboratory for Insulation Research, Massachusetts Institute of Technology, December, 1946.

barium titanate or barium-strontium titanate could be used to amplify electrical signals. The most obvious way of using a nonlinear capacitor is in a simple tuned circuit tuned to some high frequency. A low frequency signal voltage is used to vary the nonlinear capacitance. The resulting voltage across the tuned circuit is a modulated high frequency voltage. After detection the low frequency may be recovered. This arrangement is shown in Fig. 1.

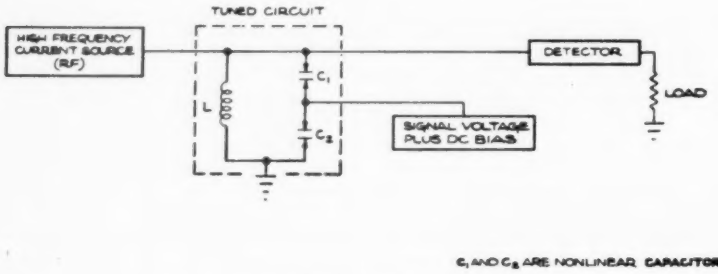

FIG. 1. Ferroelectric amplifier block diagram.

The tuned circuit is tuned to a frequency close to that of the RF source. The meaning of tuning with the nonlinear elements will be discussed below, but it might be mentioned here that it is possible to peak the high frequency voltage, if the high frequency is varied, just as in an ordinary tuned circuit. The tuned circuit will show this peak at only one frequency. Any higher harmonics are much too small in amplitude to be noticeable.

The work to be described falls into three parts:

1. Determination of dielectric properties of the materials to be used.
2. Mathematical analysis of circuit to be used.
3. Experimental investigation of actual ferroelectric amplifier.

The loss properties and the effect of temperature were not investigated.

DIELECTRIC PROPERTIES

The materials under investigation were barium-strontium titanates. For the purpose of this report, the necessary dielectric properties can be determined from a plot of electric charge *versus* applied voltage. The method for obtaining this plot is the method of Sawyer and Tower.[3] This method obtains an oscilloscope plot; the schematic is shown in Fig. 2. By making $C_1 \gg C_x$, one obtains a plot whose vertical displacement is proportional to the charge on C_x and whose horizontal displacement is, except for a small correction, proportional to the voltage across

[3] C. B. SAWYER AND C. H. TOWER, "Rochelle Salt as a Dielectric," *Phys. Rev.*, Vol. 35, February 1, 1930, p. 269.

C_x. The necessary formulas are derived in Appendix A. The surface density of charge and the electric field strength in the dielectric are then easily obtained. A typical plot is shown in Fig. 3. The exact composition of the material was unknown, but it was a mixture of barium and strontium titanates.

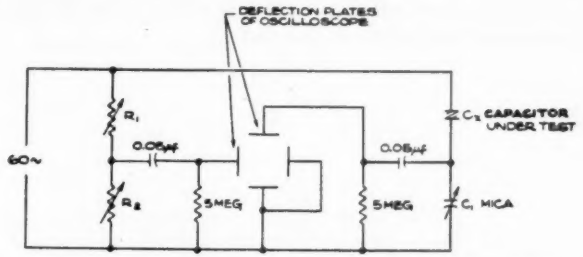

FIG. 2. Circuit for obtaining oscilloscope plot of capacitor charge *versus* capacitor voltage.

The electric displacement in a dielectric is measured by the surface density of free charge. Thus Fig. 3 is a plot of electric displacement *versus* electric field strength in the dielectric. To treat these properties analytically, it is desirable to have an equation relating D, the displacement, to E, the electric field strength. The equation that will be ob-

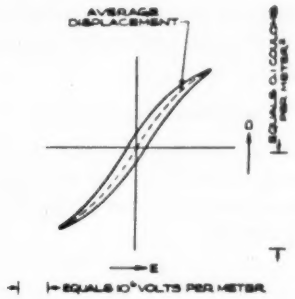

FIG. 3. Oscilloscope plot of charge density *versus* field strength for a ferroelectric substance.

tained will be one for the average displacement, represented by the dotted line of Fig. 3. As Roberts [2] points out, if one attempts to obtain an equation in the form

$$D = f(E) = aE + bE^3 + \cdots \tag{1}$$

many terms will be necessary. However, if the form

$$E = f(D) = \alpha D + \beta D^3 + \cdots \tag{2}$$

is used, two terms are sufficient for a good approximation. Therefore, we have

$$E = \alpha D + \beta D^3. \tag{3}$$

The two important quantities we want are the dielectric coefficient K, and the relative change in K per unit change in electric field strength. Naturally, both of these will be functions of E. The dielectric coefficient for a nonlinear dielectric is defined by

$$K \equiv \frac{dD}{dE}. \tag{4}$$

Using Eq. 3,

$$K = \frac{dD}{dE} = \frac{1}{\frac{dE}{dD}} = \frac{1}{\alpha + 3\beta D^2} \tag{5}$$

$$\frac{1}{K}\frac{dK}{dE} \equiv N = \frac{dK}{KdD}\cdot\frac{dD}{dE} = \frac{dK}{dD} = -\frac{6\beta D}{(\alpha + 3\beta D^2)^2}. \tag{6}$$

For the material used in the amplifier to be described below, and whose D–E curve appears in Fig. 3, the following relation was derived by substitution of two points of Fig. 3 into Eq. 3:

$$E = (1.87 \times 10^7)D + (141 + 10^7)D^3 \tag{7}$$

where D is given in coulombs per square meter, and E is given in volts per meter. From Eq. 7, we may derive the curves for K and N. These are given in Figs. 4 and 5.

MATHEMATICAL ANALYSIS

A nonlinear capacitor is distinguished by the fact that the charge on the capacitor is not proportional to the voltage across it. As in the linear case, we take the capacitance as proportional to K. Instead of the capacitance being given by

$$C = \frac{q}{v} \tag{8}$$

we have, using Eq. 4,

$$C \equiv \frac{dq}{dv} \tag{9}$$

where q is the instantaneous charge and v is the instantaneous voltage. The instantaneous current is

$$i = \frac{dq}{dt} = \frac{dq}{dv}\cdot\frac{dv}{dt} = C(v)\frac{dv}{dt} \tag{10}$$

indicating that C is a function of v.

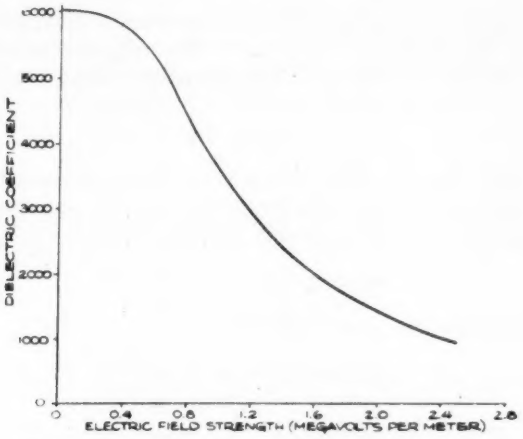

FIG. 4. Dielectric coefficient *versus* electric field strength.

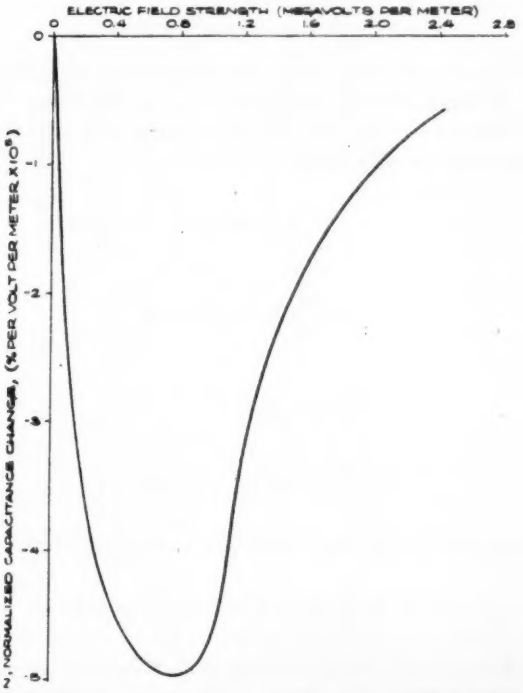

FIG. 5. Normalized capacitance change *versus* electric field strength.

Let us consider the case in which two currents of different frequencies are applied to the nonlinear capacitor. The relation between total instantaneous charge, q, and total instantaneous voltage, v, may be expressed by

$$v = \alpha_1 q + \alpha_3 q^3 \tag{11}$$

by analogy with Eq. 3. The charge may be considered to consist of two terms: one due to the high frequency, q_r, and one due to the low (signal) frequency, q_s. It is further postulated that $f_r \gg f_s$. Then

$$v = \alpha_1(q_r + q_s) + \alpha_3(q_r + q_s)^3. \tag{12}$$

Terms of like degree in q_r are collected:

$$\begin{aligned} q_r^0 &: \alpha_1 q_s + \alpha_3 q_s^3 \equiv \beta_0 \\ q_r^1 &: \alpha_1 + 3\alpha_3 q_s^2 \equiv \beta_1 \\ q_r^2 &: 3\alpha_3 q_s \equiv \beta_2 \\ q_r^3 &: \alpha_3 \equiv \beta_3. \end{aligned} \tag{13}$$

Equation 12 becomes

$$v = \beta_0 + \beta_1 q_r + \beta_2 q_r^2 + \beta_3 q_r^3. \tag{14}$$

At this point it is assumed that we may confine our attention only to the components of fundamental frequency, f_r, as providing the only significant contributions. In Eq. 14 the second and fourth terms contribute components of frequency f_r. Let

$$q_r = \frac{I_M}{\omega_r} \sin \omega_r t \tag{15}$$

so

$$i_r = \frac{dq_r}{dt} = I_M \cos \omega_r t. \tag{16}$$

Then

$$\begin{aligned} q_r^3 &= \frac{I_M^3}{\omega_r^3} \sin^3 \omega_r t \\ &= \frac{I_M^3}{\omega_r^3} \left[\tfrac{3}{4} \sin \omega_r t - \tfrac{1}{4} \sin 3\omega_r t\right]. \end{aligned} \tag{17}$$

The components of Eq. 14 which have the fundamental frequency, f_r, are

$$v_1 = \beta_1 \frac{I_M}{\omega_r} \sin \omega_r t + \tfrac{3}{4}\beta_3 \frac{I_M^3}{\omega_r^3} \sin \omega_r t. \tag{18}$$

Now, from Eq. 13, $\beta_3 = \alpha_3$, a constant. However, β_1 is not a constant; it depends upon the signal. Thus, Eq. 18 consists of a fixed amplitude term and a term whose amplitude depends on the signal

voltage. The term of constant amplitude is of no interest because no signal intelligence can be recovered from it. On the other hand, the first term does depend on the signal voltage and it is this component of the fundamental, f_r, which is of interest; this is termed v_r:

$$v_r = \beta_1 \frac{I_M}{\omega_r} \sin \omega_r t. \tag{19}$$

We must now find the meaning of β_1. If only the signal voltage is applied to the nonlinear capacitor, the signal voltage, v_s, will be determined from

$$v_s = \alpha_1 q_s + \alpha_3 q_s^3. \tag{20}$$

Then

$$\frac{dv_s}{dq_s} = \frac{1}{C(v_s)} = \alpha_1 + 3\alpha_3 q_s^2 \tag{21}$$

from Eq. 9. But comparison with Eq. 13 shows that the right hand side of Eq. 21 is β_1. This allows us to write Eq. 19 as

$$v_r = \frac{I_M}{\omega_r C(v_s)} \sin \omega_r t. \tag{22}$$

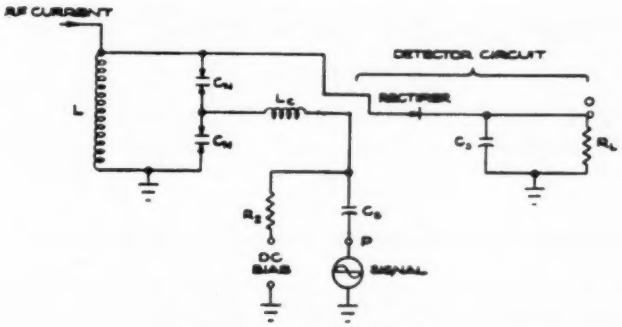

FIG. 6. Ferroelectric amplifier.

In Eq. 22 $C(v_s)$ is the nonlinear capacitance as determined solely by the signal voltage. From Eq. 22 it is seen that the impedance offered to the fundamental current is of the same form as that offered by a linear, constant capacitance, with the capacitance determined by the slope of the charge *versus* voltage curve of the capacitor when only signal frequency is applied.

The above discussion lends plausibility to the technique of treating the single-tuned circuit of Fig. 1 as an ordinary impedance as far as the high frequency (RF) source is concerned, with the capacity controlled by the signal voltage.

The circuit to be considered is shown in Fig. 6.

The capacitors labelled C_n are the nonlinear capacitors. The tuned circuit consisting of L in parallel with the C_n's in series is tuned slightly off resonance for the RF carrier frequency. C_b is a coupling capacitor and keeps d.c. out of the signal supply. R_2 is a high resistance to eliminate currents in the d-c. bias supply. L_c is an RF choke and also serves to tune the input circuit. The amplitude of the RF voltage which appears across the tuned circuit will depend upon the impedance. This impedance is varied by applying a signal to the nonlinear capacitors to change their capacitance. The result is an amplitude modulated wave which is demodulated by the detector and applied to the load resistance, R_L. As far as the RF current is concerned, the two nonlinear capacitors are in series; as far as the signal voltage is concerned, the two capacitors are in parallel. Assuming that these are equal, $C' = 4C$, where C' is the capacitance seen by the signal, and C is the capacitance seen by the carrier.

The impedance seen by the RF fundamental is given by

$$Z = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} \tag{23}$$

where ω is the RF fundamental angular frequency. If we let

$$x = R\left(\omega C - \frac{1}{\omega L}\right) \tag{24}$$

then

$$Z = \frac{R}{1 + jx}, \quad \text{and} \quad |Z| = \frac{R}{\sqrt{1 + x^2}}. \tag{25}$$

The amplitude of the varying RF voltage, V_{1M}, is given by

$$V_{1M} = I_{1M}|Z| = \frac{I_{1M}R}{\sqrt{1 + x^2}} \tag{26}$$

where I_{1M} is the constant amplitude of the current input. Define V_N by

$$V_N = \frac{V_{1M}}{I_{1M}R}. \tag{27}$$

Thus,

$$V_N = \frac{1}{\sqrt{1 + x^2}}. \tag{28}$$

If V_N is plotted *versus* x, the response curve shown in Fig. 7 results.

Operating conditions are chosen so that the static operating point (the value of x with no signal voltage) is on the side of the response curve. When a signal voltage is applied, C varies, causing x to vary,

and the operating point moves up and down on the response curve. The output of the tuned circuit will then be an amplitude modulated RF voltage. Our interest is in the amplitude of the envelope as compared to the signal voltage. For small signal voltages, the envelope may be considered to be sinusoidal, and will be a reproduction of changes in V_N. We shall speak of the envelope as the envelope voltage, V_e.

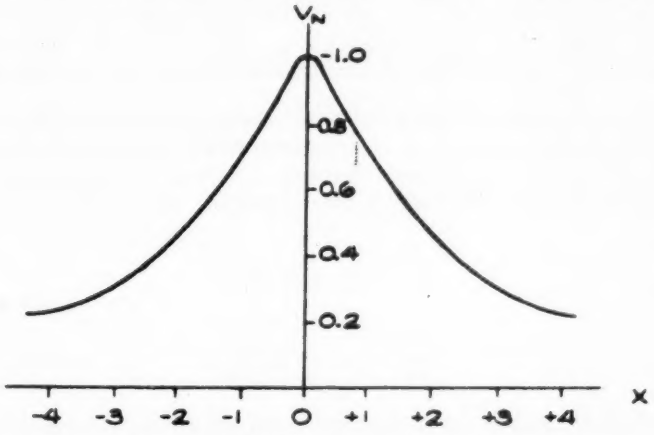

FIG. 7. Relative voltage response curve.

To find the relation between envelope voltage and signal voltage, we start with changes in V_N. Approximating small changes by differentials, we may write

$$dV_N = \frac{dV_N}{dx} \cdot \frac{dx}{dC} \cdot \frac{dC}{dv_s} dv_s \tag{29}$$

where v_s is the instantaneous signal voltage. Now, $v_s = sE$, where s is the thickness of the dielectric in the (flat plate) capacitor, and E is the instantaneous electric intensity in the dielectric.

$$\frac{dC}{dv_s} = \frac{dC}{dC'} \cdot \frac{dC'}{dv_s} = \tfrac{1}{4} \frac{dC'}{C'dE} \cdot \frac{C'}{s} = \tfrac{1}{4} N \frac{C'}{s} \tag{30}$$

since $C' = 4C$. N is the normalized capacitance change discussed under the section on dielectric properties. From Eq. 24

$$\frac{dx}{dC} = R\omega. \tag{31}$$

Thus,

$$dV_N = \frac{dV_N}{dx} \frac{R\omega C' N}{4s} dv_s. \tag{32}$$

Defining the RF circuit Q by

$$Q \equiv \frac{R}{\omega L'}, \tag{33}$$

Eq. 33 becomes

$$dV_N = \frac{dV_N}{dx} \frac{Q\omega^2 LC'N}{4s} dv_s. \tag{34}$$

The factor $\frac{dV_N}{dx}$ is the slope of the response curve, and conditions are adjusted so that operations are carried out on a steep part of this curve. When too far from resonance, the response will be very small, and if too close to resonance, distortion will result. To find the maximum value of this function, we differentiate Eq. 28 twice. The result is

$$\frac{dV_N}{dx} = -\frac{x}{(1 + x^2)^{3/2}}, \tag{35}$$

and

$$\frac{d^2V_N}{dx^2} = -\frac{1 - 2x^2}{(1 + x^2)^{5/2}} \tag{36}$$

from which we obtain the facts that the maximum slope occurs at $x = \pm \frac{1}{\sqrt{2}}$ and that the maximum slope is $0.385 \approx 0.4$. It will be assumed that the variation from this point will be small, so that $\frac{dV_N}{dx}$ remains constant at 0.4. From Eq. 27

$$dV_N = \frac{dV_{1M}}{I_{1M}R}. \tag{37}$$

The amplitude of the envelope, V_{eM}, is the change in the amplitude of the RF voltage. That is,

$$V_{eM} = dV_{1M} = I_{1M}R dV_N. \tag{38}$$

Putting these into Eq. 34, there is obtained

$$V_{eM} = (0.1)I_{1M}RQ\omega^2 LC' \left(\frac{N}{s}\right) V_{sM} \tag{39}$$

where $V_{sM} = dv_s$, for small changes. Now

and

$$\left.\begin{aligned} V_{eM} &= \sqrt{2}V_e \\ V_{sM} &= \sqrt{2}V_s \end{aligned}\right\} \tag{40}$$

if sinusoidal quantities are assumed. Further, let

$$\left.\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \\ g &= \frac{1}{R} \end{aligned}\right\}. \tag{41}$$

Then

$$V_s = (0.4)\frac{I_{1M}Q\omega^2 N}{g\omega_0^2 s} V_g. \tag{42}$$

The shunt conductance seen by the RF tuned circuit consists of two terms: the equivalent shunt conductance, g_i, representing internal losses in the tuned circuit, and the external conductance represented by the load conductance, g_L. The equivalent load as far as the RF tuned circuit is concerned is $2g_L$.[4] That is,

$$g = g_i + 2g_L. \tag{43}$$

The output power is

$$P_0 = V_s^2 g_L. \tag{44}$$

We also have

$$Q = \frac{1}{g\omega L} = \frac{\omega_0}{g\omega_0 L\omega} = \frac{\omega_0 C}{g}\left(\frac{\omega_0}{\omega}\right) = \frac{\omega_0 C}{2g_L + g_i}\left(\frac{\omega_0}{\omega}\right). \tag{45}$$

$$\therefore P_0 = (0.16)\left[\frac{I_{1M}\omega CN}{s}\right]^2\left[\frac{g_L V_g^2}{(2g_L + g_i)^4}\right]. \tag{46}$$

We consider, now, the input circuit. This is essentially a series tuned circuit with inductance L_c, capacitance C', generated voltage V_g, and circuit Q given by

$$Q_s = \frac{\omega_s L_c}{R_s} \tag{47}$$

where R_s represents the resistance. The voltage applied to each nonlinear capacitor is

$$V_s = Q_s V_g \tag{48}$$

and the power available from the signal source is

$$P_a = \frac{V_g^2}{4R_s} = \frac{V_s^2}{4\omega_s L_c Q_s}. \tag{49}$$

The power gain is given by

$$G_0 = \frac{P_0}{P_a}. \tag{50}$$

[4] F. E. TERMAN, "Radio Engineering," New York, McGraw-Hill Book Co., Inc., 1947, p. 504, footnote 2. This assumes that the efficiency of rectification is 100 per cent.

Putting Eq. 49 into Eq. 46, there is obtained

$$G_0 = (0.64)\left[\frac{I_{1M}\omega CN}{s}\right]^2\left[\frac{g_L\omega_s L_c Q_s}{(2g_L + g_i)^4}\right]. \tag{51}$$

The power gain is proportional to the Q of the signal input circuit. Increasing Q_s increases the power gain, but decreases the bandwidth. The characteristics of the dielectric enter into Eq. 51 through the factor N, which depends on how sharply the D–E curve bends over into saturation. The sharper the knee, the greater is the factor N.

It is of interest to find the value of load conductance which gives maximum power gain. Treating G_0 as a function of g, Eq. 51 may be written as

$$G_0 = K_1\frac{g_L}{(2g_L + g_i)^4}.$$

To find the maximum value of G_0:

$$\frac{\partial G_0}{\partial g_L} = \frac{(2g_L + g_i)^4 - 8g_L(2g_L + g_i)^3}{(2g_L + g_i)^8} = 0$$

from which

$$g_L = \frac{g_i}{6}. \tag{52}$$

EXPERIMENTAL FERROELECTRIC AMPLIFIER

The circuit of Fig. 6 was constructed. The RF current was supplied by a 6AC7 pentode rather than by magnetic coupling with another coil. This arrangement provides a larger RF current. The resistance in the bias circuit was several megohms and C_B was one microfarad. L was variable between 3 and 7 microhenries. The RF frequency and L were adjusted to match the capacitance of the nonlinear capacitors. The frequency chosen was 2.06 megacycles per second. Similarly, L_c was adjustable to tune the signal voltage with the nonlinear capacitors. R_L was held constant at 20,800 ohms. C_3 was held at approximately 0.002 microfarads. If C_3 was made too small, considerable RF appeared in the output. The bias voltage was adjusted to give the greatest output. This bias voltage corresponds to the electric intensity which produces maximum N (see Eq. 51). For this particular case it was 195 volts. The output voltage was examined with a cathode ray oscilloscope. All voltages were kept below the point of noticeable distortion in the output waveform. The performance of this amplifier is shown in Table I.

The amplifier described above was the last of several constructed in the Philco Laboratory. Each successive amplifier had some change to increase the power gain. The power gain of 62 was the highest ob-

tained. No close quantitative check of the theory was made, but Eq. 51 was verified qualitatively. For example, the choice of 2.06 megacycles per second was that frequency which gave maximum output. Any variation resulted in a smaller output. Furthermore, bringing the RF frequency close to the resonance point resulted in distortion of the output waveform. The fact that there is a maximum value of N, controlled solely by the d-c. bias voltage, was verified. Much remains to be done, however, in obtaining quantitative checks.

Table I shows that both voltage gain and power gain were obtained. It was thought possible, then, to make some sort of feedback arrangement so that the signal generator could be removed and have the circuit act as an oscillator. The simplest arrangement was tried first: point O was connected directly to point P in Fig. 6. Success was achieved at the first attempt. Sustained oscillations with no noticeable distortion were obtained.

TABLE I

Signal	*Volts RMS*	*Output*	*Volts RMS*	*Output Power Micro-watts*	*Power Gain*
	0.042		0.47	26.4	13.6
	0.116		1.95	181	12.2
	0.200		7.07	2400	54.6
	0.300		11.3	6130	62.0
	0.385		14.1	9560	59.2

Noticeable distortion beyond this point

Signal frequency = 17.6 kc/sec.
Input circuit Q = 15
Load resistance = 20,800 ohms
RF frequency = 2.06 megacycles/sec.
Temperature = 32° C.

The frequency of oscillation could be varied continuously from 20.8 kc per second to 32 kc. This was done by varying L_c and C_3; changing R_2 had very little effect on the frequency. Variation of the RF frequency or the amplitude of RF current had no effect on the frequency of oscillation, but did affect the amplitude of oscillation. The range of RF frequency for sustained oscillation was small.

Acknowledgments

The author wishes to thank A. C. Munster and C. T. McCoy of Philco for their sympathetic supervision of this project, and Mr. Stanley Zebrowitz of Philco, and Messrs. John Bonaccorsi and Albert Van Horn, formerly of Philco, for their assistance on the experimental phase of this research.

APPENDIX A

Let

e = instantaneous supply voltage in Fig. 2
e_x = instantaneous voltage across C_x
e_1 = instantaneous voltage across C_1
e_v = instantaneous voltage across vertical deflecting plates
e_h = instantaneous voltage across horizontal deflecting plates
s_v = vertical deflection
s_h = horizontal deflection
q = instantaneous charge on C_x or C_1 (leakage through the 5 megohm resistance is neglected). K_h and K_v are the horizontal and vertical calibration constants, respectively, in volts per unit of displacement.

Now

$$e = e_1 + e_x$$

$$e = e_v = \frac{q}{C_1}$$

$$q = C_1 e_v = C_1 K_v s_v.$$

Also

$$e_h = e\frac{R_2}{R_1 + R_2} = K_h s_h$$

$$q = K_v s_v C_1$$

and

$$e_x = K_h s_h \frac{R_1 + R_2}{R_2} - K_v s_v.$$

The correction term is $K_v S_v$.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

MEASURING PARTICLES IN THE RANGE 0.005 TO 10 MICRONS

Knowing, or being able to measure, the size of minute particles is important in many industrial and scientific fields. For example, the size of soil particles influences the capacity of the soil to absorb and retain moisture. In metallurgy the size of the ore fractions must be known in order to apply the usual separation techniques. In the paint industry the covering ability of paint is related to the size of the pigment particles. For the manufacture of abrasives and grinding wheels, both the particle-size distribution and the average particle size are important. It has also been found that soft iron will make a good magnet if the particles of that iron are small enough. The surface area of a catalyst, which depends on particle size, is often a controlling factor in the rate of a reaction. With these and many other applications, it is easy to see why methods of measuring particle size have become so important.

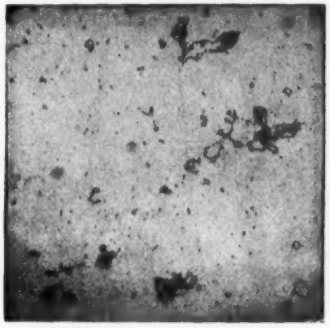

FIG. 1. Electron micrograph of 0.03 micron iron particles. Chain formations reflect magnetic character of particles.

During the past year, the Solid State Physics Division of The Franklin Institute Laboratories has been investigating methods for determining the sizes of microscopic and submicroscopic particles. Methods for measuring particles in the range 0.5 to 10 microns are well known, but less work has been done on particles as small as 0.005 micron. A single measuring technique cannot of course be used for the entire range of 0.005 to 10 microns; four methods, in fact, are used at the Laboratories.

For measuring larger particles, between 0.1 and 10 microns, an

optical microscope is used. The powder containing these particles is placed on a slide and the average diameter is determined by measuring a large number of particles, one at a time.

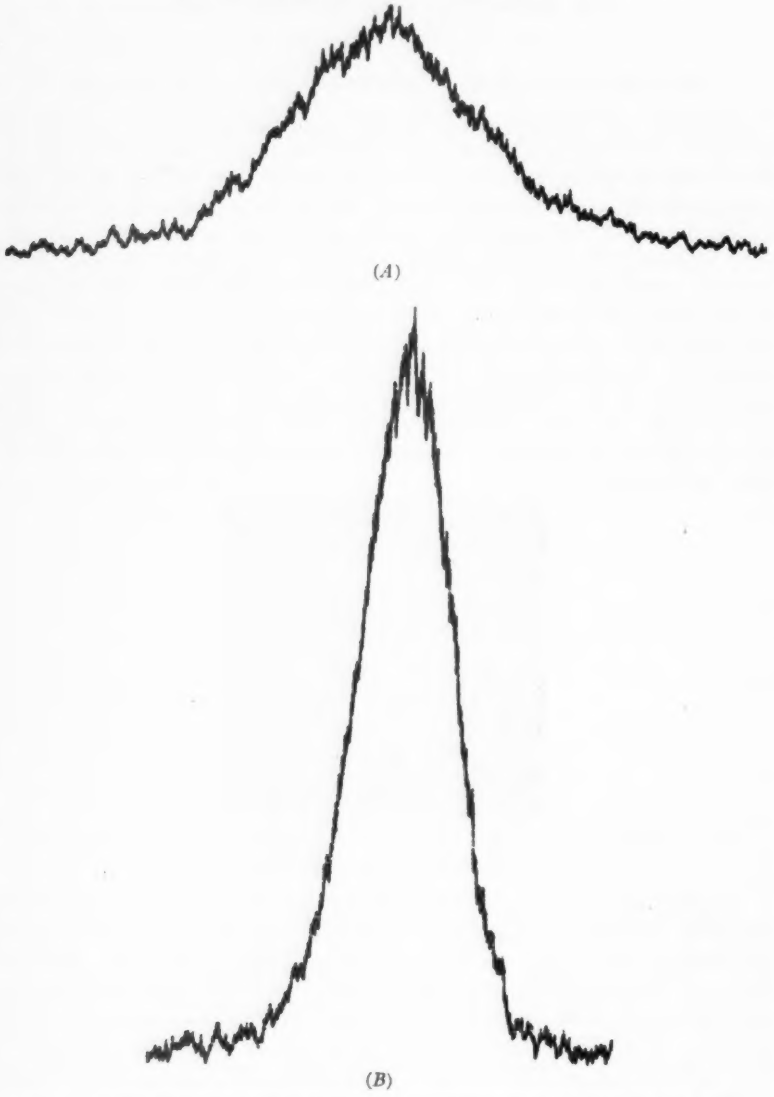

FIG. 2. X-ray diffraction lines with breadths corresponding to (*A*) 0.03 micron, and (*B*) 10.0 micron iron powder.

For smaller particles, 0.005 to 1.0 micron, the electron microscope is used. The electron microscope differs from the ordinary microscope in that it illuminates the sample with an electron beam rather than a light beam. By virtue of the much smaller wave length of electrons, much smaller particles can be resolved. A photographic record (micrograph) is made of the magnified sample, and the particles are measured directly. One difficulty with both this method and optical microscopy is that single particles must be distinguishable. Smaller particles, especially of magnetic material, cling together quite tenaciously and other methods, indirect methods, are more easily used.

X-ray diffration is one of these indirect methods. When an X-ray beam strikes a material, "reflection" takes place only at certain angles, giving rise to what are known as diffraction lines. For large crystals, these lines might be quite sharp, but for crystals below 0.08 micron in diameter, the lines become broad and diffuse. When the natural breadth of the line due to other causes has been accounted for, the diameter of the crystals in the sample may be calculated from the breadth of these diffraction lines. Where each particle contains but one crystal, this diameter is the diameter of the particle. Otherwise, where particles contain more than one crystal, another measurement must be made to determine the number of crystals per particle.

The fourth method for determining particle size is the measurement of the amount of gas, for example nitrogen, that is absorbed on the surface of the sample. With this method, particles of 0.005 to 0.5 micron may be measured. The amount of gas needed to cover the sample completely with a layer one molecule thick is measured, as well as the mass of the sample and its density. The particle size can then be calculated. This method is applicable for all sizes of particles, the only restriction being that the sample must contain sufficient surface area so that monolayer coverage corresponds to a measurable amount of gas. Two square meters of surface area is generally the minimum for accurate measurement.

Both nitrogen-adsorption and X-ray diffration techniques are valid for measuring samples whose particles are clumped and not separately distinct. Because the ranges of particle size that can be measured with these four methods overlap, it is often possible to cross-check measurements made by (say) the optical microscope and electron microscope, or the electron microscope and nitrogen adsorption, etc. Franklin Institute scientists using these techniques have made valuable contributions to many research and development projects, and increased the ability of the Laboratories to accept projects of greater variety and scope, both for industry and for the government.

R. B. CAMPBELL

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

HIGH-FREQUENCY CALIBRATION OF MAGNETIC MATERIALS

The National Bureau of Standards has recently established a calibration service for determining the r-f permeability and loss factor of magnetic materials in the frequencey range between 50 kc. and 30 Mc. The primary calibrating standard is a coaxial line of variable length constructed to dimensions of extremely high accuracy. The characteristics of magnetic materials, such as the ferrites and powdered irons, are determined in terms of the variation in the length of the line. The Bureau has also developed a secondary standard of calibration that may be simply reproduced and utilized in the manufacture of magnetic materials.

The increased use of magnetic materials in high-frequency electronic equipment has resulted in a need for more accurate evaluation of their behavior at the elevated frequencies. Ferrites and powdered irons have contributed to the reduction in size of electronic components. These materials have been modified to perform such functions as magnetic memories and amplifiers, attenuators, filters, and loop antennas for small radio receivers. In some applications, the losses in the material are of little importance, in others, high lossess are actually used to advantage; however, in most instances, it is desirable to use a material with high permeability and low loss factor.

To evaluate the characteristics of magnetic materials, experiments have been conducted on thousands of compositions, with a variety of results. Little difficulty is encountered in determining the direct-current characteristics of the materials; these experiments are performed with comparative ease and accuracy and yield valuable information. However, the procedures are not valid for determining the behavior of ferrites and powdered irons at high frequencies. These high-frequency characteristics are best described by the r-f permeability and loss factor (complex permeability) of the magnetic materials. The determination of these factors, besides being of interest to the designer and engineer, is important to the manufacturer of magnetic materials, who must use this knowledge as a basis for the quality-control phase of the manufacturing process.

The NBS primary standard of r-f permeability and loss factor was designed by P. H. Haas of the Bureau's high frequency standards laboratory. The method of measurement depends on the change in inductance of an accurately machined coaxial line when a sample of

* Communicated by the Director.

magnetic material is inserted. The variable-length coaxial line is accurately calibrated in nineteen 1-in. stops and includes a micrometer system that permits the measurement of variations to within 0.0005 in. along the 20-in. range. All machining tolerances were held to ±0.0002 in. The line is made completely from non-magnetic materials to reduce the possibility of extraneous magnetic fields affecting the measurement.

In a typical investigation, a sample of magnetic material is ground into the shape of a coaxial disk that completely fills a section of the space between the conductors of the coaxial line. In the grinding process, all tolerances are ±0.0002 in. The output terminal of the coaxial line is connected to the "unknown" terminal of an r-f bridge suitable for measuring inductance in the frequency range at which the material will be used. First, the bridge is balanced with no sample in the line and the coaxial segment extended to almost its fullest length. The disk of magnetic material is then placed on the center conductor of the line. A metal cap holding the disk in place short-circuits the end of the coaxial line. The resulting bridge unbalance is adjusted to the original conditions by a combined manipulation of the resistance reading arm on the bridge and a reduction in the length of the line. The variation in length is directly proportional to the permeability of the magnetic material relative to air and constitutes a primary method of measurement. The difference in resistance readings is a measure of the loss factor of the material and is limited only by the inherent accuracy of the bridge instrument.

The use of such highly precise and delicate calibration equipment as part of a production line, however, is restricted by the rigors of the manufacturing process. The method more commonly used in routine measurements employs a toroidally shaped ring upon which a number of windings have been placed. Quality control is achieved by comparing representative products with this "standard." The major disadvantage of this method, aside from the necessity of winding a large number of turns on each toroid to be tested, is the poor accuracy resulting from shunting capacitances and displacement currents, especially in the case of materials with high dielectric constants. Thus, a device has been needed that would give a reasonably accurate indication of the degree of variation of the finished product with respect to the "standard." The secondary standard of calibration developed by the National Bureau of Standards is designed to yield the necessary information and is rugged enough to become an integral part of the production line. Its accuracy is limited only by the precision of the bridge instrument used in the calibration.

The r-f permeameter, as the secondary standard is called, is a modification of an instrument described by G. A. Kelsall,[1] for the measure-

[1] G. A. KELSALL, "Permeameter for Alternating Current Measurements at Small Magnetizing Forces," *J.P.S.A.* and *R.S.J.*, No. 8, Feb. 1924 (pp.329–338).

ment of permeability alone at low audio and power frequencies. The principle of operation depends on the change in input impedance reflected into the primary of a transformer by load variations in the secondary. The transformer of the NBS permeameter is composed of a reference toroid and a length of coaxial line; changes in the secondary are produced by the insertion of a toroid of magnetic into the coaxial line.

The NBS r-f permeameter, also developed by P. H. Haas, is machined from 2-in. brass rod into a cylinder with an inner diameter of $1\frac{1}{2}$ in. and a length of about $1\frac{3}{4}$ in. Both ends of the line are fitted with shorting covers. The top cover is readily removable to admit the sample materials. It was made of beryllium copper so that the bearing surfaces could be machined into a springy yet electrically tight fit. The bottom cover is attached to the cylinder by screws and has a central spike, $\frac{3}{16}$ in. in diameter, which forms the center conductor of the coaxial line. A hole is tapped into this cover to receive a type N cable plug. A shelf made of low-loss insulating material separates the upper third of the cylinder from the lower portion.

The coaxial line is designed to receive toroidal rings with inner diameters greater than $\frac{3}{16}$ in., and outer diameters less than or equal to $1\frac{1}{2}$ in. A wall thickness of $\frac{1}{4}$ in. was chosen to improve the high-frequency shielding and to give ample mechanical strength. The shelf acts as a holder for the material to be tested.

A toroidal ring is slipped over the center conductor and rests on the bottom cover of the coaxial line. A number of windings, suited to the conditions of the experiment, are wound on the toroid. The material of the toroid has a low dielectric constant and a high Q at the frequency to be used during the test. One end of the winding is connected to the type N cable plug, and the other end to the coaxial line, thus making the system electrically unbalanced. In this position, the toroid acts as a reference for the measurement by becoming the primary of a transformer, the secondary of which is the metallic portion of the coaxial line.

A connection is made between the type N cable plug and a radio-frequency bridge or a Q-meter. The bridge is first balanced with the secondary of the "transformer" unloaded. The magnetic material to be tested (formed into a toroid end without windings) is then placed on the shelf in the line, and the impedance bridge is rebalanced. The variation in input impedance, as read on the bridge, is the quantity from which the permeability and loss factor of the test ferrite or powdered iron are computed.

A simple adaptation of the r-f permeameter has been made to allow the measurement of the temperature coefficient of magnetic materials. This property is important in most military applications of ferrites and powdered irons and in devices depending on elevated temperatures as a means of insuring high stability.

ISOTOPIC METHOD DETERMINES WATER CONTENT

A method which uses heavy water to determine the total water content of biological tissues and other materials has recently been developed by Drs. H. J. Morowitz and H. P. Broida of the National Bureau of Standards in connection with a program of basic instrumentation sponsored at NBS by the Department of Defense and the Atomic Energy Commission. Based on a spectroscopic measurement of the ratio of ordinary to heavy water in a solution containing the sample, the new method[1] is outstanding in the rapidity and convenience with which it can be applied to a large number of samples. Recent experiments at the Bureau have shown that it saves considerable time in determining the water of hydration of inorganic crystals and proteins, the water content of the human body and of bacterial cells. Other applications to which the technique has been applied include studies of water exchange in resins and of water transport through human capillaries, determination of moisture in the atmosphere, and the identification of unknown organic molecules.

Essentially the NBS technique consists in dissolving a known amount of the material to be analyzed in a mixture of deuterium oxide (heavy water) and hydrogen oxide (ordinary water) and then determining spectroscopically the resulting change in the ratio of deuterium oxide to hydrogen oxide. From the difference in this ratio before and after addition of the sample, the water content of the sample material, which is assumed to contain no deuterium oxide, is computed.

The analysis takes advantage of the wavelength separation of the emission lines of hydrogen and deuterium due to the isotopic shift.[2] By means of a high-frequency electrodeless discharge, water vapor from the sample is dissociated into H and OH and into D and OD. The ratio of excited hydrogen to deuterium, which is a function of the ratio of hydrogen oxide to deuterium oxide, is then determined by measuring the relative intensities of the H_β line (4861.3 Å) and the D_β line (4860.0 Å).

An entire analysis can be carried out in less than one-half hour. The sample of the water-containing material is first accurately weighed. It is then mixed with a volumetrically determined quantity of a hydrogen oxide–deuterium oxide mixture containing about 99 per cent of deuterium oxide in accurately known ratio to the hydrogen oxide. After equilibrium has been reached, vapor from this solution is continuously pumped through a discharge tube, where a steady electrodeless discharge is maintained by a 150-megacycle power supply. A

[1] For further details, see "A Rapid Spectroscopic Determination of Total Water Content," by Harold J. Morowitz and Herbert P. Broida, *Analytical Chemistry* (1952) (in press).

[2] "Optical Spectrophotometric Analysis of Hydrogen-Deuterium Mixtures in Presence of Air," Herbert P. Broida and Gerry H. Morgan, *Analytical Chemistry*, Vol. 24, p. 799 (1952).

rotating grating monochromator[3] scans the emitted radiation at a rate of about 10 Å per minute, focusing the H and D lines on a photomultiplier tube which activates a recording device. From the trace thus produced, the relative intensities of the two lines are quickly and accurately determined. By the use of an electrodeless discharge and continuous pumping of the gas through the discharge tube, sorption of hydrogen and deuterium within the tube is virtually eliminated as a source of error.

If the molar ratio of the hydrogen oxide to deuterium oxide in the original solvent is r, and the final molar ratio of the H_2 to D_2O in the solution is R, then the fraction by weight of the hydrated sample which is water, f, is given to a first approximation by the following relation:

$$f = \frac{(R - r)\, m}{(1 + r)\, M}$$

where m is the weight of the solvent in grams and M is the weight of the original material to be analyzed in grams. The approximation, which is good to about four parts in 1000, is introduced by assuming that the partial molar volumes of H_2O and D_2O are the same, and that the density of water in the range of room temperature is 1.000.

The NBS method is accurate to 0.5 per cent in the determination of water content and is applicable to any material which contains a small, well defined number of labile hydrogen atoms. Efforts are now being made at the Bureau to extend the technique to a wide range of materials.

[3] A high-resolution photoelectric grating monochromator was loaned to NBS on a field-trial arrangement by Leeds and Northrup Co.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING
November 19, 1952

The Stated Monthly Meeting of The Franklin Institute was held on November 19, 1952, in the Lecture Hall. Henry B. Allen, Executive Vice-President and Secretatary, called the meeting to order at 8:15 P.M. Approximately 255 persons attended.

Dr. Allen stated that the Minutes of the Stated Meeting in October were printed in full in the November issue of the JOURNAL. No corrections or additions being submitted, they were approved as printed. He stated further that a full account of Medal Day proceedings on October 15th will be published in the December issue of the JOURNAL.

The following elections to membership since April 30th were reported:

Active	181
Associate	256
Student	118

with a total Institute membership of 6,551 as of October 31, 1952.

It was also announced that the following have been members of the Institute for twenty-five consecutive years and have received Silver Cards of Membership:

Mr. William DeKraft, New York City
Mr. Leslie Griscom, Philadelphia

Dr. Allen then thanked Westinghouse Electric Corporation for this opportunity to present "Energy in Action" to Institute members and guests and introduced Mr. Emery Loomis, Vice-President of Westinghouse Electric Corporation. Mr. Loomis spoke briefly about the worthwhile service rendered by the Institute, both as a scientific museum and in its research laboratories.

The lecture-demonstration, "Energy in Action," illustrated low-temperature research, a turbo-jet engine designed for guided missiles, and an interesting explanation of the principles of color television. Other interesting experiments portrayed the uses of energy in various forms.

The meeting adjourned at 9:35 P.M. with a rising vote of thanks to Westinghouse Electric Corporation.

HENRY B. ALLEN
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MOON, PARRY AND DOMINA EBERLE SPENCER: Theory of the Photic Field.
MOON, PARRY AND DOMINA EBERLE SPENCER: Some Applications of Photic Field Theory.
SILBERSTEIN, LUDWIK: Elementary Probability of Efficient Absorption of a Quantum Proportional to the Volume of a Grain.
KU, Y. H. Nonlinear Analysis of Electro-Mechanical Problems.
ARSOVE, MAYNARD G.: The Algebraic Theory of Linear Transmission Networks.
MANDEVILLE, C. E.: Slow Neutron Induced Radioactivities of Nuclear Explosions: III.
BOLIE, VICTOR M.: Powerless Glides Along Zero Curvature Flight Paths.
CHANG, CHIEH-CHIEN, BOA-TEH CHU AND VIVIAN O'BRIEN: Asymptotic Expansion of the Whittaker's Function $W_{k,m}(z)$ for Large Values of k, m, z.
LANGHAAR, HENRY L.: On Torsional-Flexural Buckling of Columns.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, November 12, 1952.)

HALL OF THE COMMITTEE,
PHILADELPHIA, NOVEMBER 12, 1952.

MR. HOWARD STOERTZ in the Chair.

The following report was presented for final action:

No. 3243: Xerography.

This report recommended the award of an Edward Longstreth Medal to Chester F. Carlson, of Fairport, New York, "For his effective combination of known principles of optics, photoconductivity and electrostatics into a workable process for producing facsimiles of written matter, drawings and other documents by a system that is rapid, dry, permanent and also capable of placing a reproducible image on an offset paper master."

JOHN FRAZER,
Secretary to Committee.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 15, 1952

ACTIVE FAMILY

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ACTIVE NON-RESIDENT WITH FAMILY PRIVILEGES

Charles S. Quillen

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Charles C. Webb

NECROLOGY

John T. Bennett, Jr. '46
John K. Desmond '38
Henry J. Ettelson '43
Gordon A. Field '51
Joseph W. Hoover '42
Amelia R. Justi '37
Settembre Mammarella, M.D. '45
W. E. Moore '01
M. M. Price '11
Girard Rosemblum '43
Chao Tze Shih '47
L. S. Tarleton '42
Harry T. Woolson '32

MUSEUM

It is impossible to say when, in the age of man, guiding lights first came into use to aid the mariner. We might with some justification claim that Homer alludes to them (see Pope's translation of *The Iliad*, XIX : 405–9). Virgil also speaks of a light on Mount Leucas. The first light of undoubted record was the famous Pharos of Alexandria, built about 285 B.C. The lighthouse at Corunna, Spain is believed to be the oldest in existence, having been built in the reign of the Emperor Trajan and reconstructed in 1634. The Corduan Tower, at the mouth of the Gironde, in the Bay of Biscay, is probably the most renowned of its kind. Completed in 1611, it stands 197 feet high. The tower itself contains a chapel and numerous apartments, although the light-keeper lives in a circular structure which serves as a breakwater at the base.

The lighthouse system of this country commenced with its commerce. There is little doubt that the colonists recognized the necessity for beacons with which to guide their home-returning shallops to a safe anchorage, and that they took effective means to show the English and Dutch ships which should make their landfall at night the safe way to their harbor. The first authentic evidence of this being done at the public charge was in 1673 when it was recorded in the proceedings of the general court of the Province of Massachusetts Bay that the citizens of Nantasket (now Hull) had built a beacon on Point Allerton, the most prominent headland near the entrance to Boston Harbor, and furnished it with "fier bales of pitch and ocum." The first lighthouse was built at the entrance to Boston Harbor, on Little Brewster Island, in 1715–6. Not until 1789 did the United States authorities accept jurisdiction over the lighthouses on the coast, which at that time were nine in number.

In spite of complaints that the lighthouse service was both inadequate and inefficient very little was done to improve the service until the middle of the 19th century, and two prominent members of The Franklin Institute, Professors Joseph Henry and Alexander Dallas Bache were largely responsible for the introduction of scientific methods in illumination and fog signals. One of the most significant improvements was the introduction of the Fresnel lens. When this type of lens was first introduced into this country after its general adoption in Europe, the members of The Institute appointed a committee to examine its claims. They were most enthusiastic in endorsing the adoption of the lens for lighthouses.

The Museum has several specimens of Fresnel lenses, but much the most impressive is that which was used in the Fire Island Light between 1858 and 1932. This great lens, or more correctly group of lenses, stood 166 feet above sea level and cast its beam over nineteen miles of water. Originally its light was furnished by an oil wick lamp, but this was changed to an incandescent oil vapor lamp of 280,000 candle-power. When the lens, which may be seen on the Ground Floor of the Museum, was in service, it was operated by a clockwork mechanism and an elaborate system of weights which ran down a tube through the center of the structure. These weights required winding every four hours. It is now operated by a tiny electric motor. The whole lens weighs between four and five tons.

When it was about to be replaced in 1933, it was still in good condition, but local navigators pointed put that the arrangements for flashing were not satisfactory. The beam shone with full strength for five seconds before sweeping on to leave a fifty-five seconds period of darkness. The duration of the beam was too short to get satisfactory bearings. The result was that the new light was regulated to have eight cycles a minute, with only momentary intervals of darkness.

Visitors occasionally comment upon holes and cracks in the thick glass and wonder how these could have been caused in an isolated location and at such an altitude. A former keeper of the light told us that these were caused by sea gulls, driven before the blasts of heavy gales, colliding with the glass.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

RECENT ADDITIONS

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The Franklin Institute, a privately endowed institution which exists today only because of the generous benefactions of its supporters, is justly proud of the dividends in human benefits accrued from the investments made through the years by its friends. The accumulation of many bequests for general endowment and income to be used where it will do the most good, is the most important and pressing need of The Franklin Institute today.

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BOOK REVIEWS

MODERN PHYSICS, by F. W. Van Name, Jr. iii + 360 pages, diagrams, 16 × 23 cm. New York, Prentice-Hall, Inc., 1952. Price, $5.50.

Never before, in his long history, has man experienced such an accelerated growth in the knowledge of physical sciences as he has during the beginning of this century. Aided by highly refined mathematical tools, rapid progress has been made in every field of endeavor. The turmoil caused by this avalanche of discoveries creates a need for an occasional period of rest—a period of concentration upon past achievements during which time a consolidation of the position gained becomes possible. While the research continues, the expository résumé enables laymen and students alike to develop a clear concept of their position in relation to our physical world.

Modern Physics was written with such thoughts in mind. It is a brilliant exposé of the gains made in the field of physics during the first half of this century. In a series of well-organized chapters the author reveals himself to be a connoisseur of his subject matter. His fluid style, his keen observation of drama in historical developments, the careful analysis of sequential material chosen for presentation, are likely to appeal to his readers and recreate within them the moments of supreme delight that the study of theoretical physics promises its adepts. The author also hits upon several mathematical high spots, such as operators and eigenvalues while dealing with the state function and the Schrodinger Quantum Mechanical Operator. But he keeps his promise which prefaces the book: the reader needs very little preparation beyond the calculus. This may entice many who don't care for the rigorous mathematics of specialized papers to turn to this volume.

The book is divided into three equally strong parts. Important concepts are usually introduced early, such as Einstein's Special Theory of Relativity in the first part of the book dealing with electrons. The second part treats atoms and molecules. A fine introduction to quantum mechanics is followed by chapters on X-rays, molecular structure and spectra, and low temperature phenomena. The third part of the book has to do with nuclei and nuclear particles. Quite logically, natural and induced nuclear reactions are treated separately in preparation for the inevitable bomb. There are also some interesting speculations regarding the bomb mechanism, the size of the critical mass, and the hydrogen bomb.

The author's aim, to present us with an up-to-date picture of modern physics, is well accomplished. He has selected, very effectively, three important fields for the subject of his book. A well-documented list of supplementary readings, given at the conclusion of each chapter, serve to make professor Van Name's volume a valuable reference as well as a good text. This reviewer recommends the book highly to those who wish to gain more than a superficial view of modern physics.

CARL HAMMER

SOLUBLE SILICATES, Vol. 1, by James G. Vail. American Chemical Society Monograph No. 116. 357 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $9.00.

Considering the unspectacular nature of the subject, the authors of the book have done an admirable job of presentation. It is quite readable and not uninteresting. This first volume of a two-book monograph traces the beginnings of soluble silicates in some detail, dating from the early Phoenicians. Early and present processes in the manufacture of the soluble silicates are described with voluminous references. The fusion process involving alkali carbonate is discussed in some detail with a description of the actual changes taking place during the heating cycle. The effect of temperature on the reaction rate is shown graphically for the sodium carbonate silica mixtures.

Two other methods are described. These include the sulfate alkali and direct reaction (salt and silica) which is as yet impractical. The former process is used in Europe where the sulfate enjoys a favorable price advantage relative to the carbonate. The problems involved in dissolving the soluble glass are discussed, as are the various solid and liquid soluble silicates that are commercially available.

The major part of the volume is devoted to a discussion of the homogeneous and complex systems of soluble silicates. It is suggested that correlation exists between alkali silicate glasses and the glass of commerce, and these investigations into the physical properties of homogeneous systems of soluble silicates may constitute a starting point for studying more complex systems. Such important physical properties as viscosity, optical properties, elasticity, are gone into in detail. Actually these chapters on both the homogeneous and heterogeneous systems are so full of facts, graphs, and tables that the limited space available to the reviewer makes it difficult to do this volume justice.

The chapter on complex systems describes the effects of various groups of metals on silicate solutions, applications, gel systems, wetting action, and detergency.

It is altogether a book of considerable interest to the engineer whose work involves the use of any of the many soluble silicates available to industry.

SAMUEL N. MUCHNICK

THE RATIONAL AND THE SUPERRATIONAL, by Cassius Jackson Keyser. 259 pages, port., 15 × 21 cm. New York, Scripta Mathematica, 1952. Price, $4.25.

The name of the author of this book will be known principally to mathematicians, but the book's contents deserve to make him more widely known, for it is an excellent group of essays which reveal C. J. Keyser in philosophic vein. In this age of specialists a scientist rarely wanders from his special studies, but Keyser was not afraid to think broadly and deeply. He seems to have taken for his theme the words of Hilbert: "The infinite! No other great question has ever moved the spirit of man so powerfully; no other has stimilated his intellect so fruitfully."

Through the diligence of thinking men from time immemorial, there has been an unfolding of an understanding that infallible laws interlock all phenomena of the universe; that there is nothing of which we are conscious which is not obedient to these laws. As each advancing year is marked off the calendar of time society becomes progressively elevated to a closer communion with the power which governs the laws and life. Keyser was not only a convinced believer in this unity, but he carried his convictions a step further to a belief that the rational life was not irreconcilable with the superrational. Through his penetrating eyes the reader perceives the way by which a higher order of knowledge may be glimpsed, although never fully seen.

In the first of these essays we perceive a scientist calmly reviewing the relations of science and religion, and finding nothing incompatible. He warns us that something more than reason is required to bridge the apparent abyss between the modern concept of science and religion, the ancient mistress of science. He accepts what he terms "emotional knowledge" as the supplementary requirement, and armed with this the thinker may proceed to understanding. Having invoked the aid of emotional knowledge to assist reason, he shows, in the second of these thoughtful essays, how science may be employed to reinforce theology, to the end that the latter may be restored to its lost foremost place in intellectual studies.

In the "Significance of Death" he reaches into the subrational, for he declares "To be immortal a life must be eternal—it must be a life that never was born." This essay does not seek to prove that man is immortal. It is concerned exclusively with the relation of human death to the life that precedes it, and as a man of courage may view it.

In his essay "Thinking on Thinking" he describes autonomous thought or postulational thinking, outlines its course of development as a scientific method, exemplifies the character and variety of its achievement, asserts its availability for all fields of thought, and emphasizes its importance. Keyser's address to the American Association for the Advancement of Science in 1936 had for audience a body of philosophical students (in other words, mathematicians), yet to read it under the title of "The Role of Infinity in the Cosmology of Epicurus" is an enjoyable experience, for the cosmology of Epicurus, whether we like it or not, is an immense system of philosophic or scientific thought which is not unworthy of consideration in our day. The essay would serve its purpose if it did no more than encourage scientists to read Lucretius, whose "blending of science and song certainly has no equal in all literature." The author

sympathetically reviews the doctrine of the early atomists in the light of modern (1936) science.

For the scientifically inclined who do not reject the relationship of the rational with the superrational there is much in this book which is rewarding. As the reflection of a philosophic mind it is filled with charm, for Keyser has a literary style which might be commended to science writers.

T. C.

DESIGN FOR A BRAIN, by W. Ross Ashby. 260 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.00.

Dr. Ashby's book is an effort to explain (by means of a mechanistic model) the ability of the brain to produce adaptive behavior. In gist, the model is as follows. A cerebral control system of great complexity which arrives at a stable position by randomly exploring successive phase space fields is postulated. Stable adaptation results because unstable phase space fields result in behavior discontinuities which shift the behavior to successively different fields until stability is finally achieved. The underlying differential equations for the behavior are assumed to be linearized.

Much effort is devoted in the early chapters to describing the scientific method for studying empirical phenomena. It is a pity that the operational techniques advocated by Bridgman have not been taken to heart by Ashby. Certainly it is of interest to write down families of differential equations in multi-dimensional space for the purpose of describing behavior, but this is nothing but a mental exercise unless the scheme is intimately associated with an experimental program. To borrow from physics and consequently to overmathematize or perhaps inadequately mathematize a problem is to risk writing pseudo-equations unless the rigorous principles of operational thinking are applied throughout the thesis. It is not sufficient to demonstrate that a scheme is capable of producing certain of the characteristics of adaptive behavior, the scheme and its various postulates must be capable of proof or disproof by experimental techniques.

As far as I can tell, Ashby has not presented his thesis in such a manner that experimental correspondence between mathematical model and nature can be readily examined.

The following selection indicates one of the instances where a desire for logical rigor without regard to the realities of experimental evidence results in the author's adopting a debatable position.

"The last century's discoveries have warned us that the universe may be inexhaustible in surprises, so we should not attempt to define the environment by some formula such as 'that which obeys the law of conservation of energy,' for the formula may be obsolete before it is in print" (p. 179).

This reviewer disagrees with the foregoing quotation since it is his belief that there is no better way to investigate the unknown than to start with presently known theory and to elaborate until the elaboration or basic theory is inconsistent with experimental evidence. Science progresses from known to unknown, not from unknown to unknown.

In general, Ashby has written a stimulating book, with an interesting mathematical appendix, but his scientific orientation must be examined with great care.

EZRA KRENDEL

INSTRUMENT ENGINEERING, Vol. I, by C. S. Draper, W. McKay and S. Lees. 269 pages, diagrams, 22 × 29 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $6.00.

Instrument Engineering is one of the McGraw-Hill Publications in Aeronautical Science. It presents a generalized method of attack on the problems of measurement and control. It is a handy reference for the practicing engineer and an excellent textbook for students familiar with college physics and differential equations.

This publication was prepared from notes used over a period of twenty years in teaching both undergraduate and graduate students in the Aircraft Instrument Section and the Instrumentation Laboratory of the Aeronautical Engineering Department at Massachusetts Institute of Technology. The methods presented are based on theory from the fields of math-

matics, physics, aeronautical engineering, mechanical engineering, electrical engineering and chemical engineering, resolving into an organized pattern the background knowledge and procedures required to solve the commonly occurring measurement and control problems.

Instrument Engineering is presented in three volumes. The first, subtitled "Methods for Describing the Situations of Instrument Engineering" (or alternately, "Fundamentals"), develops a system of concepts, notations, diagrams and mathematical forms for describing the performance of physical devices. It then provides a description of the physical circumstances under consideration by means of diagrams, mathematical forms, curves and numerical data. Over one hundred figures and tables make the principles easy to visualize. Definitions, derivations and design performance are summarized for review and easy reference.

The authors carefully build up the characteristics of open-chain and closed-chain systems with reference to familiar instruments. Separate analyses are made for static and dynamic performance of instruments. A separate chapter on applications of statistical methods to instrument engineering gives particular attention to the study of probability theory used in the instrument laboratory.

The notation is that represented by "self-defining symbols." The symbols, including subscripts and superscripts, are formed from characters found on the standard typewriter. This restriction to the use of one alphabet necessitates the use of symbols with an extraordinarily large number of subscripts, sometimes as may as twelve characters. Although the system is precise, it normally requires re-education and results in lengthy equations.

At the time of this writing, Volumes II and III have not been released by the publisher. The second volume is entitled "Methods for Associating Mathematical Solutions with Common Forms" and completes the theoretical study and analysis of instruments and gives general examples of application. The third volume, "Applications of Instrument Engineering Method," bases its examples on meters, measuring system components, control system components, computing system components, regulators, servomechanisms, and feedback-control systems.

Instrument Engineering is a sound contribution to the field which it covers. It should be most helpful to anyone able to overcome the psychological hazard introduced by the "self-defining" symbols which have not been universally accepted.

A. DONALD HAY

PICTORIAL ASTRONOMY, by Dinsmore Alter and Clarence H. Cleminshaw. 296 pages, illustrations, plates, 21 × 27 cm. New York, Thomas Y. Crowell Co., 1952. Price, $4.50.

In any planetarium much material is collected, printed and distributed to the lay public over the course of years. This represents a popular elementary course in astronomy and because of type of audience this material must be presented in its most engaging and palatable form. Somehow or other this finds its way into book form and *Pictorial Astronomy* is this type of book.

Dr. Dinsmore Alter, director of the Griffith Planetarium and Observatory, and his associate Dr. Clarence H. Cleminshaw have collaborated in collecting and editing the articles which have appeared in the monthly magazine, *The Griffith Observer* to form the basis of this book.

Pictorial Astronomy is not a text book of astronomy in the formal sense. To begin, it is profusely illustrated with photographs and drawings which go far in aiding the understanding. There is also generous use of tables to give handy information concerning the astronomical bodies. As an example: when the earth is discussed there is a complete page of data to introduce our planet and there is an unusual completeness to this compilation of data. The reviewer, as a practising astronomer with many source books available, still finds this one of the most useful in his library.

The book is divided into eight sections which deal with The Sun, The Earth, The Moon, Eclipses, The Planets, Comets and Meteors, Stars and Nebulae and finally a Miscellaneous section.

The authors accepted a tremendous challenge in undertaking this work for astronomy books are many in number. Their success is assured with *Pictorial Astronomy*.

I. M. LEVITT

LINEAR ALGEBRA AND MATRIX THEORY, by Robert R. Stoll. 272 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $6.00.

The author has prepared this book as a text for an advanced undergraduate or first year graduate course in linear systems. One of the chief virtues of the book is that it is admirably suited to bridge the gap between ordinary undergraduate mathematics and completely abstract mathematics such as one finds in a course in "modern" algebra. Thus, for example, a determinant-free method for solving a system of linear equations is given. Then the notion of field is defined, and it is shown that the method can be used for the case of a system of linear equations over any commutative field.

Vector spaces and their subspaces are defined and their properties are developed, followed by the notion of isomorphism of vector spaces. All the theorems concerning determinants are developed from the postulational point of view. Bilinear, quadratic and Hermitian forms are treated as representatives of functions on a vector space into its field of scalars.

Linear transformations from one vector space to another vector space (over the same field of scalars) are discussed, and when bases are chosen the transformation is represented by a matrix. When the set of all non-singular linear transformations on a vector space are considered, the notion of group arises quite naturally. Similarly, the discussion of the set of all linear transformations on a vector space leads to the concept of ring.

Thus such modern algebraic concepts as equivalence relation, group, ring, field are treated as they naturally arise in the study of linear spaces and matrices. However, in addition to presenting the modern algebra underlying matrix theory, this book is the most comprehensive treatment of linear algebra and matrix theory on this level with which this reviewer is acquainted. The book is highly recommended. H. PLATZER

MECHANICS OF MATERIALS, by Alvin Sloane. 468 pages, diagrams, 16 × 24 cm. New York, The Macmillan Co., 1952. Price, $5.50.

No doubt it is inevitable that books on strength of materials or mechanics of materials will appear from time to time regardless of how many already exist. Undoubtedly, each writer believes he can present the material better than has been done previously. Unfortunately, however, there is frequently little difference between the new book and the old ones when a dispassionate comparison is made. It seems an awful waste of paper to print so many books on the same subject.

This present book is one of these books that contains nothing new on the strength of materials. Except for some novel, and questionable, arrangement of text (longitudinal shear is treated as a theory—corollary I of the basic bending theory, with the deflection theory as corollary II), it contains the same material as any number of other books. And, despite the fact that Mr. Sloane specifically mentions the necessity for clarity of expression in his preface, his book frequently falls short of clearness of presentation. In fact, on occasion it is quite stilted and pompous. Such phrases as "When we have established a command of the nature of attack with which we can meet the challenge of each violation (of the theory of beams), we shall have firm bases which may be built upon each other when multiple violations of basic theory arise within one application"; and again, "Such partnership of materials united to form composite beams are used to derive benefit from combination of the most desirable properties of each of the contributing materials." Such phrases are hardly examples of "clear, direct and satisfying prose."

Mechanics of Materials contain the usual items in such a book. Following an introduction to mechanics of materials, wherein stress, strain, riveted and welded joints and stress concentration are described, the basic flexural theory and its limitations, torsion, column theory, elastic strain energy and some mechanical properties of materials are discussed. The use of Mohr's circle for the determination of moment of inertia is also discussed. Definitely on the credit side, the book contains many practical problems to illustrate the application of the theory. E. W. HAMMER

ULTRASONIC PHYSICS, by E. G. Richardson. 285 pages, diagrams, 16 × 24 cm. Houston, Elsevier Press, 1952. Price, $5.00.

The branch of physics known as ultrasonics has undergone slow development since 1883. While the development of theories pertaining to the transmission and asborption of ultrasonic energy has kept pace with applications in fields other than physics and much has been written on these subjects, few volumes have been written describing the basic measuring tools of this branch of physics. In particular, not much unified treatment was available heretofore on the ultrasonic interferometer and its application as a precision instrument for research problems in molecular physics.

Ultrasonics is a term applied to artificially produced compressional waves at frequencies above about 20 kc/s. The subject matter is treated from the point of view of one inquiring into the construction, use, accuracy, and precision of the ultrasonic interferometer. The organization and integration of the material reflects to a great extent the work which the author and his colleagues and students have carried out over the past twenty years, as well as contemporary work in other laboratories. Much of the material is a review of published data, with specific reference to the original authors and their publications, and a considerable amount of new information which should make the volume worthy of interest to specialists for review. It is doubtful that this book could serve as a textbook, but it would have application as a general reference in this limited field.

Of the eight chapters, the opening three are devoted to descriptions of sources of ultrasonic energy, methods of detection and the physical properties of this radiation, and equipment and techniques for the measurement of velocity and propagation constants. It is for the latter purpose, where the measuring system is such that the acoustical waves are stationary, that the ultrasonic interferometer is useful. Several types of these measuring devices are described as well as some interesting applications of ultrasonic waves, such as for an optical diffraction grating.

The remaining five chapters are devoted to a rather extensive presentation of the results of research into the propagation characteristics of ultrasonic energy in gases, liquids, solids (metal plates and rods, rubber-like materials, high polymers and fibers), and dispersive systems (aerosols and hydrosols) and into theories of anomalous propagation and relaxation theory. Throughout the book are numerous sketches of special laboratory equipment required to make the measurements which are reported. The references are up-to-date, making the volume valuable from this point of view.

S. CHARP

THEORY OF VIBRATIONS, by N. W. McLachlan. 154 pages, illustrations, 13 × 19 cm. New York, Dover Publications, Inc., 1951. Price, $2.45 (cloth); $1.25 (paper).

This book is based upon a series of graduate course lectures given by the author at Brown University. The text presents an analytical study of vibrational problems and requires that the reader have an elementary working knowledge of Fourier's integral theorem, operational calculus, and Bessel functions.

In general the chapters are arranged in order of analytical difficulty starting with simple linear systems having one degree of freedom with emphasis on mechanical systems. The discussion is supplemented by electrical analogs. Forced vibrations of simple linear systems are then taken up and the concept of mechanical impedance is introduced. A study of an electromechanical system is included in this section along with the application of a rectangular impulse to a simple system and its solution by operational calculus. Nonlinear systems having one degree of freedom are next considered and abetted by an interesting discussion of intermodulation. The discussion of discrete systems is concluded with a short chapter on systems having more than one degree of freedom and the application of Lagrange's equation.

Analyses of continuous systems take up the remainder of the book. Transverse and longitudinal vibration of flexible strings are considered, followed by longitudinal and torsional vibration of uniform bars and tubes. The next chapter takes up the transverse vibration of bars and tubes with various end conditions and gives solutions for forced vibration by operational calculus. Vibrations of circular and annular membranes and circular plates are dis-

cussed and exact solutions obtained by means of Bessel functions. Sound waves of finite amplitude are treated in the final chapter.

Although a great amount of material is covered in a relatively short text the author's exposition is clear throughout. Many of the details are, however, left to the reader. Practical applications and exercises are not included. This is in accordance with the author's intention since the book was written for use as a reference by graduate students taking a formal course in the theory of vibrations. Amplification of the contents, practical applications, and exercises have therefore been left to the lecturer.

Approximate methods such as those of Rayleigh, Ritz, and Stodola, found in most engineering texts, are not included. This strictly analytical text is of special value to the practicing engineer for its application of operational calculus to vibration problems and for exact solutions of circular plates. M. P. NORIN

RADIO ASTRONOMY, by Bernard Lovell and J. A. Clegg. 238 pp., tables, drawings and illustrations, 14 × 19 cm. New York, John Wiley & Sons, Inc., 1952. Price, $4.00.

A new science, Radio Astronomy, has come into prominence since the last war. Using the techniques of radio and radar allied with the subject matter of physics, astrophysics and astronomy this new science has delved into a domain that has heretofore been allotted only to astronomy. Remarkable results have been obtained and that is the principal reason for this book. The authors have tried to present the most important aspects of this new subject in such a manner that it will be readable to as wide a circle of readers as possible.

The first two chapters offer an elementary introduction to fundamental astronomy; chapters three and four introduce some of the basic radio techniques involved. Chapters five through nine offer information about meteoric phenomena and the measurement techniques that have evolved from this new research tool with particular regard to meteor velocities. Chapters ten and eleven offer for our inspection some of the contributions to the field of physics. A sample of the topics presented is: what happens to meteoric particles as they enter the earth's atmosphere, how they do so, the mechanisms of the production of electrons and the scattering of radio waves from the electron trail Chapters 13 through 15 cover solar disturbances, radio emissions from sunspots, and solar flares as the principal topics. The six remaining chapters are devoted to galactic radio emissions, the radio stars and investigtion of the moon and planets by radio techniques.

This book is amply illustrated and a bibliography is supplied at the end of each chapter to indicate where the reader can obtain a more detailed account of the subject matter. The author's free style of writing makes this an enjoyable book to read and it is well recommended to all who have an interest in this field. ALAN F. PETTY

SUN, MOON AND PLANETS, by Roy K. Marshall. 129 pages, diagrams, 13 × 19 cm. New York, Henry Holt and Company, 1952. Price, $2.50.

After writing a book on the broad aspects of atomic energy and the universe which enjoyed a phenomenal sale, Dr. Roy K. Marshall has gone back to his first love, astronomy.

In *Sun, Moon and Planets*, the former director of the Fels and Morehead Planetaria has turned to writing about the solar system With the skill, informality and facility born of at least 5000 planetarium demonstrations, the fundamental members of the solar system are introduced, described and their motions explained.

The language throughout the book is clear, concise and understandable by the layman and the little analogies like the compairng of sunspots with measles are an index to his ability to resolve abstruse points.

Sun, Moon and Planets is a brilliant introduction to one of the most fascinating of the sciences—astronomy—and as such is highly recommended. I. M. LEVITT

ACIDS AND BASES—THEIR QUANTITATIVE BEHAVIOUR, by R. P. Bell. 90 pages, 11 × 17 cm. New York, John Wiley & Sons, Inc.; London, Methuen & Co., Ltd.; 1952. Price, $1.50.

This monograph consists of a unified account of acid-base equilibria in aqueous and non-aqueous solution. The influence of interionic attraction on acid-base equilibira as well as correlations of acid-base strength with molecular structure are very well presented.

The discussion of acid-base catalysis is concerned largely with the quantitative relationships which have been experimentally observed to govern the kinetics of reactions subject to general catalysis. Consideration of the mechanism of acid-base catalysis is brief and limited, in the case of keto-enol tautomerism, to the familiar Pedersen concept of a duality of mechanism. Complete omission of Swain's recently proposed mechanism (involving concerted electrophilic-nucleophilic attack on the substrate) is somewhat regrettable because of its possible significance to the general theory of homogeneous catalysis.

The book serves, however, as a valuable introduction to the Bronsted-Lowry concept of acids and bases and utilizes prominent experimental evidence to illustrate the various subjects under discussion.

JAMES F. ROTH

BOOK NOTES

FLEAS, FLUKES AND CUCKOOS, by Miriam Rothschild and Theresa Clay. 304 pages, plates, 16 × 22 cm. New York, The Philosophical Library, 1952. Prices, $8.75.

The unusual title of this book is highly descriptive of the contents, for fleas, flukes and cuckoos are the three groups of bird parasites treated at length by the authors, who are recognized authorities on the subject. Excellently illustrated with some 40 plates and numerous pen and ink drawings, the book is a scholarly treatise—the first in the field. As such, it should be of special interest to ornithologists and zoologists.

DESCRIPTIVE GEOMETRY, by E. G. Pare, R. O. Loving and I. L. Hill. 309 pages, diagrams, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1952. Price, $4.00.

In this new text the authors aim to fulfill the need for a descriptive geometry textbook in which "fundamentals are presented in the same pedagogically sound units of work usually introduced in laboratory periods." The text is organized along the lines of the previously published *Descriptive Geometry Worksheets* by the same authors, in which new principles are introduced in order of need and difficulty. Solutions in pictorial form are provided and emphasis is placed on applications to engineering experience.

GENERAL COLLEGE MATHEMATICS, by W. L. Ayres, Cleota G. Fry and H. F. S. Jonah. 283 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $3.75.

Intended as a textbook for a terminal one-year course in mathematics at the first year level in college, this volume is supposed to furnish a general mathematical background for students in biological sciences, social sciences and the humanities. Historical notes appear throughout, and emphasis is placed on statement of principles rather than on drill techniques and formulas. Some of the topics—such as installment buying, population studies, etc.—seem to this reviewer to be high school, rather than college level, as do the "Problems for Fun" sections in each chapter, but it is presumed that these are part of the "distinct effort . . . to capture the student's interest." The chapter headings are tricked up with "cute" subtitles, such as "Seeing is Believing," "An Equation is a Sentence is a Sentence is a Sentence," "Waves but no Water," "How to Influence People and Lose Friends," etc. One cannot help but wonder why this should be necessary for the liberal arts college student.

QUANTITATIVE CHEMICAL ANALYSIS, by Leicester F. Hamilton and Stephen G. Simpson. Tenth edition, 529 pages, diagrams, 15 × 22 cm. New York, The Macmillan Company, 1952. Price, $4.50.

First published in 1897, this standard text is now appearing in its tenth edition—which bears little resemblance to the first one. The changes in this edition are: additions to the literature references; the adoption of the Lewis convention in dealing with electrode potentials; and added sections on the principles underlying colorimetric, conductometric, amperometric, photometric and other special methods of analysis.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

2-DESOXY-D-GLUCOSE AS AN INHIBITOR IN THE AEROBIC GLUCOSE METABOLISM OF YEAST

BY

GLADYS E. WOODWARD

Previous reports from this laboratory have shown that 2-desoxy-D-glucose (2DG), a structural analogue of glucose, is a potent inhibitor of several vital processes involving glucose metabolism, namely, anaerobic fermentation by yeast (1), anaerobic glycolysis by rat tumor slices (2), and growth of embryonic chicken-heart fibroblasts (3). The inhibition of anaerobic yeast fermentation was shown to be due to competition between 2DG and glucose for an enzyme in yeast.

Further studies on the effect of 2DG on yeast metabolism have now shown that 2DG is an even more potent inhibitor of growth than of fermentation. The inhibition of yeast growth also appears to be of the competitive type. Yeast fermentation under aerobic conditions was found to be inhibited by 2DG to about the same extent as under anaerobic conditions. Yeast respiration, however, was not appreciably inhibited by 2DG.

METHODS

Yeast growth studies were carried out in 25 ml. Erlenmeyer flasks. To each flask were added 5 ml. of nutrient broth (Difco) which had been adjusted to pH about 5.5 with HCl and autoclaved. Solutions of glucose or of glucose plus 2DG were sterilized by filtration and graded amounts added to the nutrient broth to give a final volume of 6 ml. The media in the flasks were inoculated with 2 drops (0.04 ml.) of an 18-hr. culture of *Saccharomyces cervisiae* (A.T.C.C. No. 7921) grown in 6 ml. of the broth containing 2 per cent glucose in a tube at 30° C. The experimental cultures were then shaken on a horizontal shaker in an incubator at 30° C. After 27 hours the growth was measured as the packed volume of cells obtained by centrifugation.

Fermentation and respiration studies were carried out in the Warburg manometric apparatus at 37° C. with air as the gas phase (4). Respiration (O_2-uptake) was measured by the direct method in which KOH is placed in the center well of a flask to absorb the CO_2 produced. The reaction mixtures consisted of 8 mg. pressed bakers' yeast (Fleischmann's starch-free) and glucose or glucose plus 2DG in 2 ml. of 0.02 M phosphate buffer. The yeast was tipped in from the side-arm of a flask after temperature equilibration, and the rate of O_2-uptake or CO_2-

evolution measured. The CO_2 from fermentation was calculated as the CO_2 in excess of that produced by oxidation of the glucose, assuming a respiratory quotient of 1.

RESULTS AND DISCUSSION

The inhibition of yeast growth produced by graded amounts of 2DG in media containing 2 per cent or 5 per cent glucose is shown in Table I. The data presented were obtained with a pure culture of

TABLE I.—*Inhibitory Effect of 2DG on the Growth of Yeast and Reversal of the Inhibition by Glucose.*

2DG Concentration.	Inhibition of Growth* In 2% Glucose.	In 5% Glucose.
%	%	%
0.900		100
0.450		85
0.225	100	55
0.180	100	(41)**
0.090	89	11
0.045	(50)**	0
0.036	43	0
0.018	18	0
0.009	0	

* Averaged from 4 to 8 determinations.
** Interpolated.

yeast (A.T.C.C. No. 7921), and essentially the same results were obtained in the 2 per cent glucose medium when yeast from Fleischmann's yeast cake was used. Growth was completely inhibited in 2 per cent glucose by 0.18 per cent and higher concentrations of 2DG, while lower concentrations of 2DG allowed some growth. As little as 0.018 per cent 2DG produced significant inhibition of growth in the 2 per cent glucose medium. When the glucose concentration was increased to 5 per cent, considerably less inhibition was produced at each level of 2DG than was observed with 2 per cent glucose. This reversal of inhibition by increasing the glucose concentration indicates that the 2DG is inhibiting by competing with the glucose as substrate for yeast growth.

The effect of 2DG on the respiration and aerobic fermentation of yeast is shown by the data in Table II. The respiration was not

TABLE II.—*Effect of 0.02 M 2DG on the Respiration and Aerobic Fermentation of 0.02 M Glucose by Yeast*

Substrate	pH (Initial)	Respiration O_2 Absorbed.	Inhibition.	Fermentation CO_2 Evolved.	Inhibition.
		µl./hr.	%	µl./hr.	%
Glucose (no 2DG)	4.9	223		461	
Glucose + 2DG	4.9	221	1	115	75
Glucose (no 2DG)	6.8	209		288	
Glucose + 2DG	6.8	204	2	53	82

significantly inhibited by 2DG when present in equi-molar amount to glucose. The degree of inhibition of fermentation, 75 per cent at pH 4.9 and 82 per cent at pH 6.8, was almost identical with the degree of inhibition, 77 to 80 per cent, previously found under anaerobic conditions (1).

Comparison of the inhibitory effect of 2DG toward yeast growth with that toward fermentation shows that 2DG is at least 15 times more potent as an inhibitor of yeast growth than of yeast fermentaion.

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CURRENT TOPICS

Atomic-Powered Industrial Trucks.—Atomic propulsion of electric industrial trucks is one of many engineering advances in materials handling expected during the next few years, B. I. Ulinski, Director of Engineering, Automatic Transportation Company, Chicago, said recently.

Ulinski, who attended the Iron & Steel exposition in Cleveland, believes that with the present pace of progress in atomic energy research related to industrial applications, revolutionary developments in propulsion power supply can be expected.

He explained that the splitting of uranium 235 atoms creates a number of radioactive fission by-products, such as iodine, barium, zirconium, cerium, etc. These by-products are really "ashes" piling up in the waste tanks of plutonium works such as those at Hartford.

Electrical power can be generated directly from such radioactive materials, Ulinski said. "If a pure beta emitter is properly positioned and insulated in a vacuum tube, it will build up to a high voltage as it loses its beta particles (electrons)," he pointed out. "While the electric power tapped from such an experimental conversion unit is small, probably a watt, it is nevertheless useful, and it will keep on generating for years without refueling or maintenance."

Ulinski said this radioactive process of generating electric power is similar to the common photo-electric cell principle, used in such devices as light meters. In the photo-cell process, he continued, the light striking photosensitive material releases electrons from its surface, causing a current to be generated. This current, while small, is controlled by the intensity of the light striking the photosensitive material and registers its value on the light meter.

"Similarly, the current generated by a beta emitter can also power small instruments," he said.

According to Ulinski, it is not too far fetched to expect, in the coming decade, revolutionary developments in the conversion of nuclear fission products into "tremendous amounts of electrical current. Studies of the many problems connected with the use of fission products in power generation are now underway.

"Conventional storage batteries for industrial uses, such as powering electric material handling tractors, can very well be replaced by atomic conversion units," he asserted.

Ulinski also pointed out that it is already possible to convert carbon to electrical energy through the medium of heated reaction fuel coils. By proper coordination of today's scientific discoveries, it should not be too difficult to convert nuclear energy to electrical energy without resorting to intermediate mechanical means, he added.

One solution to major steel handling problems—that of handling bigger and heavier coils—will be the construction of lighter tractors, embodying straddle-type design, Ulinski said. These will be equipped with a retractable ram.

The handling of loose tinplate, he stated, will be facilitated by a magnetic holding device, replacing the usual forks and clamps combination. Such a magnetic attachment, Ulinski said, will eliminate slippage and sliding problems during transit and especially at times of brake application.

Automatic's engineering director foresees widespread use of electronic dispatching and control equipment, such as radar, radio, and television.

Ulinski declared air conditioned cabs for operator comfort and power brakes and steering equipment are possibilities in some installations, while even remote control tractors, operated by portable television and push buttons, are feasible.

Gel-Time Meter.—An ingenious new timing meter which measures and registers the exact time when certain fluid materials jell is bringing improved quality control into the production of paint components, varnishes, synthetic resins and plastics.

According to Dr. Robert E. Burnett, a chemist at the General Electric Company Research Laboratory, Schenectady, N. Y., an instrument known as the G-E gel-time meter initially was developed for laboratory staff use, but now is finding its way into plants that make plastics, paints and varnishes.

Speaking at the annual meeting of the American Chemical Society, he said that data furnished by the meter "provide vast information about the qualities of the final product while the materials are in an intermediate state," or a semi-solid condition.

He said that former laboratory methods for checking the time required for chemical reaction of materials that turn to gels were less exact and far too time-consuming. The new meter completes the entire operation automatically.

Test substances are automatically maintained at a given temperature in the apparatus while a motor-driven stirring rod slowly rotates. When increased thickening of the material intensifies the pull on the rod, a spring closes electrical contacts, stopping a visible time meter, which registers the time down to one-tenth of a second. This action also halts the machine's motor and signals the operator who has been free to prepare additional samples.

Under a given set of conditions, Dr. Burnett pointed out, the instrument will register the gel-time characteristics of a fluid undergoing these changes with variations of less than plus or minus one per cent on successive tests.

"This permits the resin manufacturer to cook his reaction mixtures to the desired extent within closer tolerances," he explained. "In fact, by determining gel-times at intervals during the cooking of a batch of resin, the progress of reaction can be closely followed."

When a batch of material is finished, he said, a gel-time measurement on the product will indicate its quality. The meter also can be used to show whether aging or contamination of products has occurred during storage, shipment or use.

Water Purification Unit.—The Army's new truck-mounted electrical water purification unit, which is capable of treating, clarifying, filtering, purifying and distributing 2400 gallons of water an hour in temperatures as low as 40 degrees below zero, was demonstrated before defense and city officials at

Richmond, Va., recently. The unit, which can easily be transported on a cargo-type plane to combat zones, contains several inventions and numerous developments resulting from years of basic and applied research at the Engineer Research and Development Laboratories, Fort Belvoir, Va.

Heart of the unit is an ultra-high rate solids contact water conditioner called an erdlator. This device is connected in series with diatomite filters, and the combination not only removes silt and destroys bacteria, but also provides positive physical barriers to the passage of organisms which cause dysentery and schistomiasis, a serious water borne disease found in the Far East.

Polluted water can be pumped into the unit and chemically and physically treated and discharged as potable water in approximately 20 minutes. Its excellent performance and short detention time are attributed to use of a material which is almost universally available and costs less than $12.00 a ton.

Only one man is required to operate the unit and it can be put into operation by three men in less than 90 minutes after arrival at a selected site. Equipped with its own power source, it can be operated in total blackout. The only noise audible is from the engine driving the generator.

Atomic Instruments Used in Oil Exploration.—"The recent introduction of atomic instruments in oil exploration has opened up a new geological era," declared James Mitchel, geophysicist of the Radiac Co., Inc., New York City. "It has cut millions of dollars from the cost of petroleum exploration and has been a boon to our defense effort. The introduction of atomic instruments into the oil industry has resulted in a tremendous drop in extravagant 'wildcatting'—the indiscriminate drilling for oil in territory not known to be productive."

The use of radioactivity detection instruments in oil exploration was an unexpected by-product of aerial surveys for uranium made shortly after World War II by several Canadian "flying physicists." However, it is believed that the Russians used this technique in the early thirties. Russian scientists have been using radioactivity detection instruments for cosmic ray studies for many years.

While prospecting for uranium from low-flying aeroplanes, the Canadians used the latest innovation in airborne atomic instrumentation, the "Scintillometer," an ultra-sensitive gamma ray detector, which they had developed. The Scintillometer is several hundred times more sensitive to gamma rays than the Geiger counter. Gamma rays are emitted from uranium in the earth. The Scintillometer measures radioactivity by electronically counting the scintillations of a grid of synthetic crystals struck by atomic radiation. The Canadian geophysicists arrived at a surprising conclusion. Scintillation counting disclosed that almost invariably around the edges of oil fields, there is a "halo" of increased radioactivity. The Scintillometer also revealed that there is less radioactivity directly above the oil fields.

These studies have been of tremendous significance in oil exploration. Indirectly, they have also led to the discovery of valuable minerals, many of which are vital to our defense program.

"The use of radioactivity detection and measurement instruments in oil exploration is somewhat analogous to military strategy," commented Mr.

Mitchel, who also directs field research for the Radiac Co., Inc. "First, there is the 'Air Force Intelligence,' locating with airborne scintillation counters the general 'halo' areas. Then, in comes the 'Motorized Infantry' carrying mounted Geiger counters on trucks, jeeps, or cars. Finally, come the 'Infantry foot soldiers,' those oil prospectors who pinpoint the 'hot' areas by close low-ground scanning with Geiger counters. Sometimes they lower a Geiger counter into a shallow two-foot hole or into a drill hole several hundred feet deep. The combined operations of prospecting teams in the air and on land have tamed prodigal wildcatting into an efficient science. The Radiac Co., Inc. has furnished more atomic instruments this year for oil and mineral exploration than for uranium prospecting, in spite of the walloping large bonus offered by the government to the successful uranium prospector."

D-C. Current-Measuring Reactor.—A d-c. current-measuring reactor which safely measures d-c. current up to 120,000 amperes has been announced by the Special Products Section of the General Electric Company. Normal accuracy is plus-or-minus two per cent, and customer specified accuracies of one or one-half per cent are also available.

In standard models of 1000, 3000, 5000, 8000 and 10,000 amperes, the new reactor is availabe up to 120,000 amperes on special order. It is designed for users of large amounts of d-c. power such as the electro-chemical, steel and aluminum industries.

The main feature of the new device is that it isolates the control leads from the current being measured. In the conventional shunt method of measuring current, the shunt leads are at the d-c. potential. With the current-measuring reactor, instrument leads are energized at low a-c. voltage, usually 120 volts, 60 cycles, reducing shock hazard materially at the control panel.

Due to electrical isolation between the primary bus and the secondary wiring, the main circuit is not affected by any faults that occur in the control wiring. The argument for using high voltage cable in the control circuits is removed, with an economic saving in some cases, according to G.-E engineers.

The leads to the instrument need not be calibrated and can be any reasonable length. Thus, control leads are not specified in advance, and can be cut to size at the time of installation. If additional instruments are desired they may be added to the secondary circuit with greater ease than is possible with the shunt system.

The new device slips in place over the d-c. bus and does not require that the bus be broken or that bolted connections be made at the point of measurement. Another advantage is that the reactor uses only a small amount of power from an a-c. source.

In operation, two toroidal-wound cores are saturated by the direct-current in the bus until the a-c. current is applied. The a. c. desaturates the core of one coil on one half-cycle and the other core on the following half-cycle. The point of desaturation limits the a-c. current. The value of this a-c. current, indicated on an ammeter, is proportional to the intensity of the d-c. saturation, thus measuring the d-c. current.

The cores are enclosed in a molded shell made of glass-reinforced polyester resin.

A new tube-coating method developed by the Naval Research Laboratories, ONR, offers important applications in the fields of nuclear physics, optics, and electricity. Most important use to date has been in production of halogen-quenched Geiger-Mueller counter tubes, with these advantages over tubes now available commercially: wider range of operating voltages, non-sensitivity to light, almost indefinite operating life, elimination of critical materials, such as copper or stainless steel. The process permits control of deposit thickness, gives a transparent, nonmetallic, electrically conducting film which may be cleaned and washed by any regular means. Although colorless, the films appear colored due to light reflection-interference patterns; colors of gray, brown, violet, blue and green have been produced by various thickness of film.

Transistors in Wide Range of Experimental Applications.—First demonstrations showing the scientific progress made towards harnessing the tiny transistor, which performs many of the functions of electron tubes, in a wide range of applications useful to radio, television, and industry were held recently at the David Sarnoff Research Center of RCA.

Transistors made from specks of germanium crystal were shown operating an experimental portable television receiver, radio sets, loudspeaker systems, miniature transmitters, parts of electronic computers, and other experimental devices, many of which are believed to be the first of their kind. Each development was in the form of a laboratory model which, it was emphasized, is still in the preliminary and experimental stage.

Appraising the present status of transistor development, Dr. E. W. Engstrom, Vice President in Charge of RCA Laboratories Division, said:

"These demonstrations highlight the fact that transistors are today no longer entirely a research concern. They are in the fields of radio and television, an immediate problem for advanced development by industry engineers who can learn how to put them to work in evolving more versatile, smaller, sturdier, and eventually lower cost equipment for industry and the public."

Dr. Engstrom said that RCA does not expect the transistor to supplant the electron tube "any more than radio replaced the phonograph. In fact, the market for electron tubes may even increase under the full impact of commercial transistors.

"This is because the transistor will allow the development of electronic devices now undreamed of," he said. "Many of these devices will still require the work of electron tubes and in quantities that will continue to tax the manufacturing capacity of the electron tube industry. Thus, as transistors begin to replace certain tubes in present electronic equipment, the displaced tubes will find new jobs in new devices made possible by the development of the transistor."

Dr. Engstrom said that the experimental equipment demonstrated represented exploratory employment of transistors in many phases of electronics and that the items were laboratory models in a research stage. Neither the transistors nor their applications are yet in commercial form.

Among the experimental equipment shown was a portable, battery-operated television receiver, tubeless except for the picture tube. In its initial

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form, it is a one-channel set with a five-inch screen. In another approach, transistors were introduced in part of the circuits of a standard television receiver. Transistors were also employed in circuits of industrial television equipment and the "Walkie-Lookie" portable TV camera equipment to point up savings in weight, size, and power consumption.

In the radio field, all-transistor AM, FM, and automobile radio receivers were demonstrated. These exploratory applications already point the way to new compactness and much lower battery drain than receivers require today. The automobile radio operating with transistors directly off the 6-volt battery of a car eliminates the relatively costly high-voltage power supply common to present-day automobile sets and enables more than a ten-fold reduction in battery drain.

Also demonstrated was a small portable 45-rpm phonograph with a self-contained transistor amplifier and battery supply. Another transistor amplifier circuit was displayed in a portable public address system with self-contained battery supply.

A transformerless transistor power amplifier, an entirely new kind of circuit that may have considerable significance in the design of radio and television receivers, was displayed and operated. The experimental power amplifier circuit consists of nothing but four transistors mounted in a small tube socket. For many audio applications it appears that such a device, which harnesses a unique transistor characteristic known as complementary symmetry, can do the job that now requires two or more tubes, an output transformer, a phase inverter, and other components to amplify audio signals into a loudspeaker.

Two transistorized sections of modern electronic computers, a counter and an adder, were operated to show how transistors can do at least as good a job as tubes in such circuits and at the same time offer great advantages in size, reliability and power consumption.

Two examples of how transistors might be practical in musical devices were also displayed. One was an eight-note transistor "piano" the oscillations from which were picked up and "played" by a standard radio receiver. The other was an electronic ukelele, working on the same principles as an electric guitar, except that the compactness and low-power requirements of transistors enable the amplifier and loudspeaker to be self-contained within the instrument itself.

Dr. Engstrom explained that the equipment made use of the ten types of RCA developmental and experimental transistors, three of the point-contact variety and seven of the junction transistor family. The three point-contact types and one of the junction types will be offered on a limited sampling basis for engineering advanced development to industry representatives who attended the demonstration. The other six junction types are still under research and are classed as experimental, he said.

Dr. Engstrom emphasized that transistors are not interchangeable with present tubes in the sense that a tube from an existing instrument can be pulled out and a transistor substituted. New circuitry must be developed, he explained, in order to take full advantage of the special characteristics of transistors and, to fully realize space and weight saving possibilities which they open up, new components will also have to be designed.

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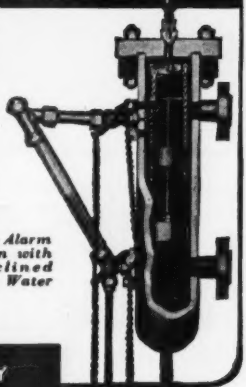

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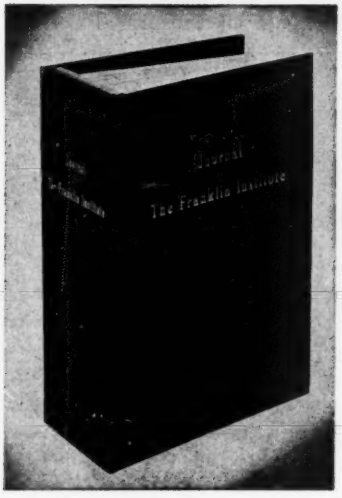

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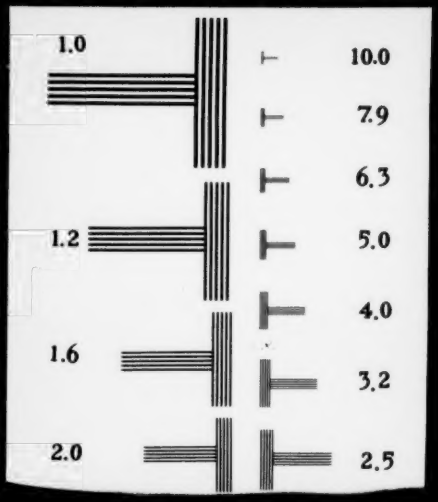

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